Radiant Mathematics-8

1. Rational Number

Exercise 1.1

1. (a) $\frac{2}{5}$

$\because \quad \frac{p}{q} = \frac{p \times n}{q \times n} \qquad (\therefore n = 1, 2, 3 \ldots\ldots)$

then $\quad \frac{2}{5} = \frac{2 \times 2}{5 \times 2}; \frac{2}{5} = \frac{2 \times 3}{5 \times 3}; \frac{2}{5} = \frac{2 \times 4}{5 \times 4}; \frac{2}{5} = \frac{2 \times 5}{5 \times 5}$

$= \frac{4}{10}, \frac{6}{15}, \frac{8}{20}$ and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

(b) $\frac{8}{-11}$

$\because \quad \frac{p}{q} = \frac{p \times n}{q \times n} \qquad (n = 1, 2, 3, 4 \ldots..)$

Then $\quad \frac{8}{-11} \quad \Rightarrow \quad \frac{8 \times 2}{-11 \times 2} = \frac{8 \times 3}{-11 \times 3} = \frac{8 \times 4}{-11 \times 4} = \frac{8 \times 5}{-11 \times 5}$

$= \frac{16}{-22} = \frac{24}{-33} = \frac{32}{-44} = \frac{40}{-55}$

(c) $\frac{-12}{13}$

$\because \quad \frac{p}{q} = \frac{p \times n}{q \times n} \qquad (n = 1, 2, 3, 4 \ldots..)$

Then $\quad \frac{-12}{13} \frac{-12 \times 2}{13 \times 2} = \frac{-12 \times 3}{13 \times 3} = \frac{-12 \times 4}{13 \times 4} = \frac{-12 \times 5}{13 \times 5}$

$= \frac{-24}{36} = \frac{-36}{39} = \frac{-48}{52} = \frac{-60}{65}.$

(d) $\frac{-5}{9}$

Rational Number $\frac{p}{q} \times \frac{n}{n}$

Then $\quad \frac{-5}{9} \quad \Rightarrow \quad \frac{-5 \times 2}{9 \times 2} = \frac{-5 \times 3}{9 \times 3} = \frac{-5 \times 4}{9 \times 4} = \frac{-5 \times 5}{9 \times 5}$

$= \frac{-10}{18} = \frac{-15}{27} = \frac{-20}{36} = \frac{-25}{45}.$

2. $\frac{p}{q}$ is a rational number $= \frac{21}{-8}$

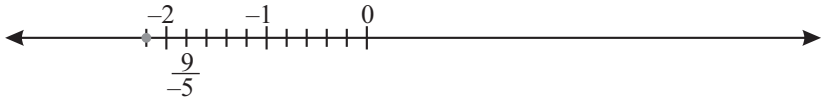
Denominator $= q$

then $\quad q = 24.$

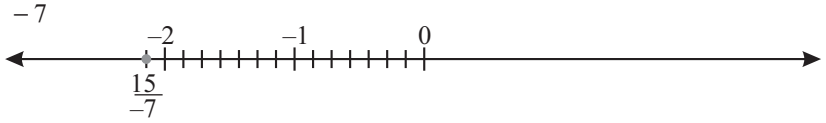
Rational number $= \frac{21 \times -3}{-8 \times -3} \qquad (\because 24 \div -8 = -3)$

$= \frac{-63}{24}.$

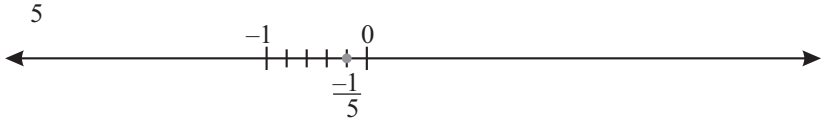
3. (a) $\frac{9}{-5}$



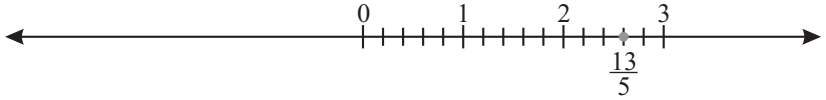
(b) $\frac{15}{-7}$



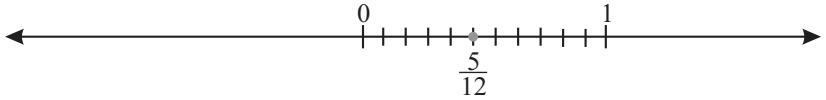
(c) $\frac{-1}{5}$



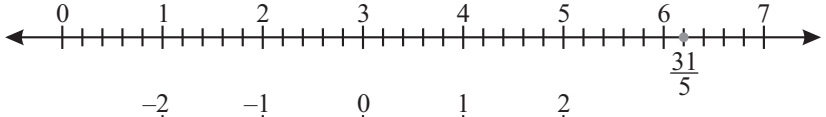
(d) $\frac{13}{5}$



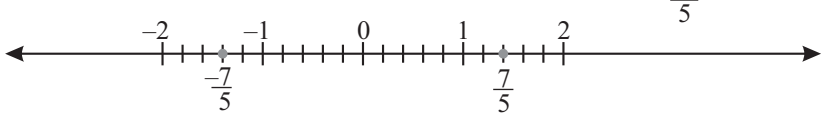
(e) $\frac{5}{12}$



(f) $6\frac{1}{5} \Rightarrow \frac{31}{5}$



4.



5. (a) $\frac{-1}{5}, \frac{-3}{10}, \frac{-11}{15}, \frac{13}{20}$

LCM of 5, 10, 15, 20 is 60

$\frac{-1}{5} \times \frac{12}{12} = \frac{12}{60}; \frac{-3 \times 6}{10 \times 6} = \frac{18}{60}; \frac{-11}{15} \times \frac{4}{4} = \frac{44}{60}; \frac{13}{20} \times \frac{3}{3} = \frac{39}{60}$

$\frac{-12}{60}; \frac{-18}{60}; \frac{-44}{60}; \frac{39}{60}$

$\frac{39}{60} > \frac{-12}{60} > \frac{-18}{60} > \frac{-44}{60}$

So, $\frac{-11}{15} < \frac{-3}{10} < \frac{-1}{5} < \frac{13}{20}$

5	5, 10, 15, 20
2	1, 2, 3, 4
3	1, 1, 3, 2
2	1, 1, 1, 2
	1, 1, 1, 1

(b) $\frac{3}{10}, \frac{-4}{15}, \frac{-15}{20}, \frac{-11}{20}, \frac{19}{30}$

LCM of 10, 15, 20, 30, is 60.

5	10, 15, 20, 20, 30
2	2, 3, 4, 4, 6
2	1, 3, 2, 2, 3
3	1, 3, 1, 1, 3
	1, 1, 1, 1, 1

$\frac{3\times 6}{10\times 6} = \frac{18}{60}$; $\frac{-4\times 4}{15\times 4} = \frac{-16}{60}$;

$\frac{-15\times 3}{20\times 3} = \frac{-45}{60}$; $\frac{-11\times 3}{20\times 3} = \frac{-33}{60}$

$\frac{19\times 2}{30\times 2} = \frac{38}{60}$

$$\frac{-45}{60} < \frac{-33}{60} < \frac{-16}{60} < \frac{18}{60} < \frac{38}{60}$$

$$\frac{-15}{20} < \frac{-11}{20} < \frac{-4}{15} < \frac{3}{10} < \frac{19}{30}$$

(c) $\frac{4}{7}, \frac{6}{-9}, 0, \frac{-2}{7}, \frac{1}{7}$

$\frac{4}{7}, \frac{-6}{9}, 0, \frac{-2}{7}, \frac{1}{7}$

LCM of 7, 9, 7, 7 is 63.

$\frac{4}{7}\times\frac{9}{9} = \frac{36}{63}$; $\frac{-6}{9}\times\frac{7}{7} = \frac{-42}{63}$, 0, $\frac{+1}{7}\times\frac{9}{9} = \frac{9}{63}$

$\frac{-2}{7}\times\frac{9}{9} = \frac{-18}{63}$

$\Rightarrow$ $\frac{36}{63}, \frac{-42}{63}, 0, \frac{9}{63}, \frac{-18}{63}$

$$-54 < -18 < 0 < 9 < 36$$

$$-\frac{6}{9} < \frac{-2}{7} < 0 < \frac{1}{7} < \frac{4}{7}$$

(d) $\frac{-3}{7}, \frac{-5}{14}, \frac{-25}{28}, \frac{-1}{2}$

LCM of 7, 14, 28, 2, is 28.

7	7, 14, 28, 2
2	1, 2, 4, 2
2	1, 1, 2, 1
	1, 1, 1, 1

$\frac{-3}{7}\times\frac{4}{4} = \frac{-12}{28}$, $\frac{-5}{14}\times\frac{2-10}{2-28} = \frac{-10}{28}$

$\frac{-25}{28}\times\frac{1}{1} = \frac{-25}{28}$ $-\frac{1}{2}\times\frac{14}{14} = \frac{-14}{28}$

$\frac{-12}{28}, \frac{-10}{28}, \frac{-25}{28}, \frac{-24}{28}$

$\frac{-25}{28}, \frac{-14}{28}, \frac{-12}{28}, \frac{-10}{28}$

$$\frac{-25}{28} < \frac{-1}{2} < \frac{-3}{7} < \frac{-5}{14}$$

6. (a) $\left|\frac{2}{3}-\frac{3}{4}\right|=\left|\frac{8-9}{12}\right|=\left|\frac{-1}{12}\right|\Rightarrow\frac{1}{12}$

(b) $\left|\frac{5}{3}-\frac{-7}{6}\right|=\left|\frac{10-7}{6}\right|=\left|\frac{3}{6}\right|=\left(\frac{1}{2}\right)=\frac{1}{2}.$

(c) $\left|\frac{-27}{25}\div\frac{3}{5}\right|\times\left|\frac{-15}{12}\times\frac{-16}{5}\right|\quad\left|\frac{-27}{25}\times\frac{5}{3}\right|=\left|\frac{-135}{75}\right|\Rightarrow\frac{135}{175}$

$\left|\frac{-15}{64}\times\frac{-16}{5}\right|\Rightarrow\left|\frac{240}{320}\right|\Rightarrow\frac{240}{320}$

then, $\frac{135}{75}\times\frac{240}{320}\Rightarrow\frac{9}{5}\times\frac{6}{8}=\frac{54}{40}\frac{54}{40}=\frac{27}{28}=\frac{27}{20}.$

7. $|x\times y|=(x)\times|y|$ if $x=\frac{3}{4}$ and $y=\frac{1}{2}$

By taking $x=\frac{3}{4}$ and $y=\frac{1}{2}$

$$|x\times y|=\left|\left(\frac{3}{4}\right)\times\left(\frac{1}{2}\right)\right|$$

$\Rightarrow\quad\left|\frac{3}{8}\right|=\frac{3}{8}$

and $|x|\times|y|=\left|\frac{3}{4}\right|\times\left|\frac{1}{2}\right|=\frac{3}{4}\times\frac{1}{2}=\frac{3}{8}$

$\therefore\quad|x\times y|=|x|\times|y|$

8. Verify that $|x+y|\le|x|+|y|$ when

(a) $x=\frac{8}{-3}$ and $y=\frac{-7}{9}$

$x=\frac{8}{-3}=\frac{-8}{3}\qquad y=\frac{-7}{9}$

$$|x+y|=\left|\frac{-8}{3}+\frac{-7}{9}\right|=\left|\frac{-24-7}{9}\right|=\left|\frac{-31}{9}\right|=\frac{31}{9}$$

and $|x|+|y|=\left|\frac{-8}{3}\right|+\left|\frac{-7}{9}\right|\Rightarrow\frac{8}{3}+\frac{7}{9}\frac{24+7}{9}=\frac{31}{9}$

$$\frac{31}{9}=\frac{31}{9}$$

Hence, $|x+y|\le|x|+|y|$ is true.

(b) $x=\frac{-9}{7}$ and $y=\frac{3}{4}$

$x=\frac{-9}{7},\qquad y=\frac{3}{4}$

$$|x+y|=\left|\frac{-9}{7}+\frac{3}{4}\right|=\left|\frac{-36+21}{28}\right|=\left|\frac{-36+21}{28}\right|$$

$\left|\frac{-15}{28}\right|=\frac{15}{28}\quad|x|+|y|=\left|\frac{-9}{7}\right|+\left|\frac{3}{4}\right|$

$$\frac{9}{7}+\frac{3}{4}=\frac{36+21}{28}=\frac{57}{28}\,\frac{15}{28}<\frac{57}{28}$$

Hence, $|x+y|\leq|x|+|y|$ is true.

9. (a) $\frac{-5}{8}\times\frac{10}{-6}$

$\Rightarrow \frac{-5\times10}{8\times-6}=\frac{-50}{-48}$ $\Rightarrow$ $\frac{-25}{-24}\Rightarrow\frac{25}{24}.$

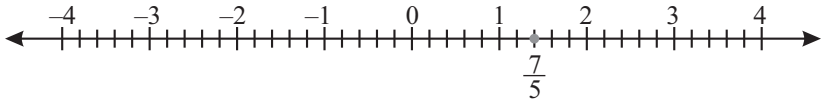
(b) $\frac{3}{7}-\frac{5}{8}$ $\Rightarrow$ $\frac{24-35}{56}=\frac{-11}{56}.$

(c) $\frac{1}{2}-\frac{1}{3}\times\frac{1}{5}\,\frac{1}{2}-\frac{1}{3}\times\frac{1}{5}=\frac{1}{2}-\frac{1}{15}=\frac{15-2}{30}=\frac{13}{30}.$

10. Since $\left|\frac{-7}{5}\right|=\frac{7}{5}$ and $\left|\frac{7}{5}\right|=\frac{7}{5}$

therefore two numbers whose absolute value is $\frac{7}{5}$ is $\frac{7}{5}$ and $\frac{-7}{5}$

these are the two equidistant number on sides of zero.



Exercise 1.2

1. (a) $\frac{-17}{12}$ and $\frac{-5}{18}$

$\Rightarrow \frac{-17}{12}+\frac{-5}{18}=\frac{-21+(10)}{36}=\frac{-21-10}{36}=\frac{-31}{36}.$

(b) $\frac{-7}{3}$ and $\frac{5}{3}$ $\Rightarrow$ $\frac{-7+5}{3}=\frac{-2}{3}.$

(c) $\frac{-13}{19}$ and $\frac{6}{19}$ $\Rightarrow$ $\frac{-13+6}{19}=\frac{-7}{19}$

(d) $\frac{-16}{6}$ and $\frac{3}{10}$ $\Rightarrow$ $\frac{-5+90}{30}=\frac{4}{30}.$

2. (a) $\frac{-7}{15}+\frac{11}{15}=\frac{-7+11}{15}=\frac{4}{15}.$

(b) $\frac{3}{14}+\frac{-6}{14}=\frac{3-6}{14}=\frac{-3}{14}$

(c) $\frac{-2}{3}+\frac{-5}{6}=\frac{-4+(-5)}{6}=\frac{-9}{6}=\frac{-3}{2}$

3. (a) $-8+\frac{-11}{12}=\frac{-11}{12}+(-8)\frac{-8}{1}+\frac{-11}{12}=\frac{-11}{12}+\frac{(-8)}{1}$

$\Rightarrow \frac{-96-11}{12}=\frac{-11-96}{12}$

$\Rightarrow \frac{-107}{12}=\frac{-107}{12}$

L.H.S. = R.H.S.

(b) $\frac{-3}{4}+\frac{-7}{8}=\frac{-7}{8}+\frac{-3}{4}$

$\frac{-6-7}{8}=\frac{-7-6}{8}$

$\Rightarrow \quad \frac{-13}{8}=\frac{-13}{8}$

L.H.S. = R.H.S.

4. Additive inverse is negative of a rational number $\frac{p}{q}$ is $\frac{-p}{q}$.

(a) $\frac{-10}{11}=\frac{10}{11}$ (b) $\frac{2}{7}=\frac{-2}{7}$ (c) $\frac{6}{17}=\frac{-6}{17}$

(d) $\frac{-18}{-7}=\frac{18}{7}$ (e) $\frac{9}{-22}=\frac{9}{22}$

5. (a) $\left(\frac{7}{2}+\frac{-4}{3}\right)+\frac{3}{5}=\frac{7}{2}+\left(\frac{-4}{3}+\frac{3}{5}\right)$

L.H.S. $\left(\frac{7}{2}+\frac{-4}{3}\right)+\frac{3}{5}=\left(\frac{21-8}{6}\right)+\frac{3}{5}=\frac{13}{6}+\frac{3}{5}=\frac{65+18}{30}=\frac{83}{30}$

R.H.S. $\frac{7}{2}+\left(\frac{-4}{3}+\frac{3}{5}\right)=\frac{7}{2}+\left(\frac{-20+9}{15}\right)=\frac{7}{2}+\frac{-11}{15}=\frac{105-22}{30}=\frac{83}{30}$

$\frac{83}{30}=\frac{83}{30}$

L.H.S. = R.H.S.

(b) $\left(\frac{-5}{8}+\frac{9}{8}\right)+\frac{13}{8}=\frac{-5}{8}+\left(\frac{9}{8}+\frac{13}{8}\right)$

$\left(\frac{-5+9}{8}\right)+\frac{13}{8}=\frac{-5}{8}+\frac{22}{8}\Rightarrow\frac{-5+22}{8}$

$\frac{4+13}{8}=\frac{17}{8}=\frac{17}{8}$

L.H.S. = R.H.S.

6. (a) If $x=\frac{-7}{18}$

$y=\frac{-4}{15}\frac{-7}{18}+\frac{-4}{15}=\frac{-4}{15}+\frac{-7}{18}$

$\frac{-35+(-24)}{90}=\frac{-24+(-35)}{90}$

$\frac{-35-24}{90}=\frac{-24-35}{90}$

$\frac{-59}{90}=\frac{-59}{90}$

L.H.S. = R.H.S.

Hence, $x + y = y + x$.

(b) If $x = \frac{-5}{12}$

$$y = \frac{2}{9}\frac{-5}{12} + \frac{2}{9} = \frac{2}{9} + \frac{-5}{12}$$

$$\frac{-15+18}{36} = \frac{8-15}{36}$$

$$\frac{-7}{36} = \frac{-7}{36}$$

L.H.S. = R.H.S.

Hence, $x + y = y + x$.

7. (a) If $x = \frac{-11}{12}\ y = \frac{-5}{6}\ z = 3$

$$\left(\frac{-11}{12} + \frac{-5}{6}\right) + 3 = \frac{-11}{12} + \left(\frac{-5}{6} + 3\right)$$

$$\left(\frac{-11-10}{12}\right) + \frac{3}{1} = \frac{-11}{12} + \left(\frac{-5+18}{6}\right)$$

$$\frac{-21}{12} + \frac{3}{1} = \frac{-11}{12} + \frac{13}{6}$$

$$\frac{-21+30}{12} = \frac{-11+26}{12}$$

$$\frac{15}{12} = \frac{15}{12}$$

L.H.S. = R.H.S.

(b) $(x + y) + z = x + (y + z)$

$x = \frac{4}{5},\ y = \frac{-3}{6}\ z = \frac{-2}{15}$

$$\left(\frac{4}{5} + \frac{-3}{6}\right) + \frac{-2}{15} = \frac{4}{15} + \left(\frac{-3}{6} + \left(\frac{2}{15}\right)\right)$$

$$\left(\frac{24-15}{30}\right) + \frac{-2}{15} = \frac{4}{5} + \left(\frac{-15-4}{30}\right)$$

$$\frac{9}{30} + \frac{-2}{15} = \frac{4}{5} + \frac{-19}{30}$$

$$\frac{9-4}{30} = \frac{24-19}{30}$$

$$\frac{5}{30} = \frac{5}{30}$$

L.H.S. = R.H.S.

8. (a) $-10 + \frac{-17}{12} = \frac{-17}{12} + - \mathbf{10}$

(b) $\left(\frac{-7}{8} + \frac{5}{-12}\right) + \frac{9}{16} = \frac{-\mathbf{7}}{\mathbf{8}} + \left(\frac{5}{-12} + \frac{9}{16}\right)$

(c) $\frac{2}{-3} + \mathbf{0} = \mathbf{0} + \frac{2}{-3} = \frac{-2}{3}$

(d) $\frac{-2}{5} + \left(\frac{7}{15} + \frac{-3}{2}\right) = \left(\frac{-2}{5} + \frac{\mathbf{7}}{\mathbf{15}}\right) + \frac{-3}{2}$

Exercise 1.3

1. (a) $\frac{-7}{12}$ from $\frac{-5}{18}$

$$\frac{-5}{18} - \left(\frac{-7}{12}\right) = \frac{-5}{18} + \frac{7}{12} = \frac{-10+21}{36} = \frac{11}{36}$$

(b) $\frac{2}{5}$ from $\frac{1}{15}$

$$\frac{1}{15} - \frac{2}{5} = \frac{1-6}{15} = \frac{-5}{15} = \frac{-1}{3}$$

(c) $\frac{-3}{7}$ from $\frac{12}{14}$

$$\frac{12}{14} - \left(\frac{-3}{7}\right) = \frac{12}{14} + \frac{3}{7} = \frac{12+6}{14} = \frac{18}{14} = \frac{9}{7}$$

2. (a) $\frac{-7}{12} - \left(\frac{-8}{15}\right) = \frac{-35-(-32)}{60} = \frac{-35+32}{60} = \frac{-3}{60}$

(b) $\frac{9}{8} - \left(\frac{-4}{5}\right) = \frac{45-(-32)}{40} = \frac{45+32}{40} = \frac{77}{40}$

(c) $\frac{-3}{8} - \left(\frac{-1}{16}\right) = \frac{-6-(-1)}{16} = \frac{-6+1}{16} = \frac{-5}{16}$

(d) $\frac{7}{42} - \frac{8}{21} = \frac{7-16}{42} = \frac{\overset{3}{9}}{\underset{14}{42}} \Rightarrow \frac{-3}{14}$

3. Let required number be x

$$\left(\frac{5}{6} - \frac{2}{3}\right) - x = \frac{-7}{18}$$

$$\left(\frac{5-4}{6}\right) - x = \frac{-7}{18}$$

$$\Rightarrow \quad \frac{1}{6} = \frac{-7}{18} + x$$

$$x = \frac{1}{6} + \frac{7}{18} = \frac{3+7}{18}$$

$$x = \frac{10}{18} = \frac{5}{9}$$

4. Suppose required number = x

$$x + \frac{-7}{2} = \frac{-8}{5}$$

$$\Rightarrow \qquad x - \frac{7}{2} = \frac{-8}{5}$$

$$x = \frac{-8}{5} + \frac{7}{2} \qquad \Rightarrow \qquad x = \frac{-16+35}{10} = \frac{19}{10}$$

5. Let number = x

$$\frac{7}{-8} - x = \frac{5}{24} \qquad \Rightarrow \qquad \frac{7}{-8} = \frac{-7}{8}$$

$$-x = \frac{5}{24} - \frac{-7}{8} \qquad \Rightarrow \qquad -x = \frac{5-(-21)}{24}$$

$$\Rightarrow \qquad x = \frac{26}{24} = \frac{13}{12}$$

6. (a) $\left(\frac{8}{15} + \frac{6}{5}\right) - \frac{5}{12} = \frac{8}{15} + \left(\frac{6}{5} - \frac{5}{12}\right)$

$$\left(\frac{8+18}{15}\right) - \frac{5}{12} = \frac{8}{15} + \left(\frac{72-25}{60}\right)$$

$$\frac{26}{15} - \frac{5}{12} = \frac{8}{15} + \frac{47}{60}$$

$$\frac{104-25}{60} = \frac{32+47}{60}$$

$$\frac{79}{60} = \frac{79}{60} \text{ Its } \textbf{true.}$$

(b) $8 - \left(2\frac{3}{5} + 2\frac{5}{12}\right) = 8 - 2\frac{3}{5} - 2\frac{5}{12}$

$$8 - \left(\frac{13}{5} + \frac{29}{12}\right) = 8 - \frac{13}{5} - \frac{29}{15}$$

$$8 - \left(\frac{156+145}{60}\right) = \frac{480-156-145}{60}$$

$$8 - \left(\frac{301}{60}\right) \frac{480-301}{60} = \frac{179}{60}$$

$$\frac{179}{60} = \frac{179}{60} \text{ Its } \textbf{true.}$$

7. (a) $\frac{5}{16} - \left(-\frac{2}{3}\right) = \frac{5}{16} + \frac{\mathbf{2}}{\mathbf{3}}$ (b) $-\frac{2}{15} - \left(\frac{-7}{12}\right) = \frac{\mathbf{-2}}{\mathbf{15}} + \frac{7}{12}$

8. (a) $x = \frac{-5}{12}\ y = \frac{2}{3}$

$$\frac{-5}{12} - \frac{2}{3} \qquad \Rightarrow \qquad \frac{-5-8}{12} = \frac{\overset{1}{3}}{\underset{4}{12}} \qquad \Rightarrow \qquad \frac{1}{4}$$

(b) $x = \frac{7}{9}\ y = \frac{-2}{3}$

$$\frac{7}{9} - \frac{2}{3} \qquad \Rightarrow \qquad \frac{7+6}{9} = \frac{13}{9}$$

Exercise 1.4

1. (a) $\frac{9}{8}$ by $\frac{2}{3} = \frac{\overset{3}{9}}{8} \times \frac{2}{3} = \frac{3}{4}$

(b) $\frac{-9}{15}$ by $\frac{-5}{21} = \frac{\overset{3}{-9}}{15} \times \frac{\overset{1}{-5}}{21} = \frac{3}{21} = \frac{1}{7}$

(c) $\frac{-9}{11}$ by $\frac{22}{23} = \frac{-9}{11} \times \frac{\overset{2}{22}}{23} = \frac{-8}{23}$.

2. (a) $\left(\frac{2}{3} \times \frac{15}{-16}\right) - \left(\frac{7}{12} \times \frac{-24}{35}\right)$

$= \left(\frac{2}{3} \times \frac{\overset{5}{15}}{-16}\right) - \frac{7}{12} \times \frac{\overset{2}{-24}}{35}$

$= \left(\frac{5}{-8}\right) - \left(\frac{-2}{5}\right) \quad \Rightarrow \quad = \frac{-5}{8} - \frac{(-2)}{5}$

$= \frac{-25-(-16)}{40} = \frac{-9}{40}$

(b) $\left(\frac{-4}{5} \times \frac{15}{8}\right) + \left(\frac{-1}{3} \times \frac{9}{7}\right) - \left(\frac{2}{9} \times \frac{27}{14}\right)$

$= \left(\frac{-4}{5} \times \frac{\overset{3}{15}}{8}\right) + \left(\frac{1}{3} \times \frac{\overset{3}{-9}}{7}\right) - \left(\frac{2}{9} \times \frac{27}{14}\right)$

$= \frac{-3}{2} + \frac{3}{7} - \frac{3}{7} = \frac{-21+6-6}{14}$

$= \frac{-21}{14} = \frac{-3}{2}$

3. (a) $\left(\frac{3}{4} \times \frac{1}{2}\right) \times \frac{5}{7} = \frac{3}{4} \times \left(\frac{1}{2} \times \frac{5}{7}\right)$

$\frac{3}{8} \times \frac{5}{7} = \frac{3}{4} \times \frac{5}{14}$

$\frac{15}{56} = \frac{15}{56}$

(b) $\left(\frac{-7}{6} \times \frac{-2}{5}\right) \times \frac{3}{8} = \frac{-7}{6} \times \left(\frac{-2}{5} \times \frac{3}{8}\right)$

$\frac{\overset{7}{14}}{30} \times \frac{3}{\underset{4}{8}} = \frac{-7}{6}\left(\frac{-2}{5} \times \frac{3}{\underset{4}{8}}\right)$

$$\frac{\overset{7}{21}}{\underset{40}{120}} = \frac{-7}{\underset{2}{6}} \times \frac{-3}{20}$$

$$\frac{7}{40} = \frac{7}{40}$$ **Proved.**

(c) $\frac{2}{3} \times \left(\frac{4}{5} + \frac{7}{8}\right) = \left(\frac{2}{3} \times \frac{4}{5}\right) + \left(\frac{2}{3} \times \frac{7}{8}\right)$

$$\frac{2}{3} \times \left(\frac{32+35}{40}\right) = \frac{8}{15} + \frac{14}{24}$$

$$\frac{\overset{1}{2}}{3} \times \frac{67}{\underset{120}{40}} = \frac{64+70}{120}$$

$$\frac{67}{60} = \frac{\overset{67}{134}}{\underset{60}{120}}$$ **Proved.**

(d) $\frac{-6}{15} \times \left(\frac{7}{8} + \frac{-5}{12}\right) = \left(\frac{-6}{15} \times \frac{7}{8}\right) + \left(\frac{-6}{15} \times \frac{-5}{12}\right)$

$$\frac{-6}{15} \times \left(\frac{7}{8} + \frac{-5}{12}\right) = \frac{-21}{60} + \left(\frac{\underset{3}{-6}}{\underset{2}{18}} \times \frac{-5}{12}\right)$$

$$\frac{-6}{15} \times \left(\frac{21-10}{24}\right) = \frac{-21}{60} + \frac{1}{6}$$

$$\frac{-6}{15} \times \frac{11}{24} = \frac{-11}{60}$$

$$\frac{-21+60}{60} = \frac{-11}{60}$$

$$\frac{-11}{60} = \frac{-11}{60}$$

4. Fill in the blanks :

(a) $-19 \times \frac{-5}{12} = \frac{-5}{12} \times -$ **19**

(b) $\left(\frac{6}{11} \times \frac{-20}{21}\right) \times \left(\frac{-7}{8}\right) = \mathbf{\frac{6}{4}} \times \left(\frac{-20}{21} \times \frac{-7}{8}\right)$

(c) $\frac{1}{8} \times \left(\frac{-2}{5} + \frac{6}{17}\right) = \frac{1}{8} \times \frac{-2}{5} + \mathbf{\frac{6}{17}}$

(d) $\frac{-21}{40} \times \left(\frac{3}{7} \times \frac{17}{-24}\right) = \left(\frac{-21}{40} \times \frac{17}{-24}\right) \times \mathbf{\frac{3}{7}}$

5. (a) $\frac{7}{4} \times \left(\frac{5}{8} + \frac{1}{2}\right) = \frac{7}{8} \times \frac{5+4}{8}$

$$= \frac{7}{4} \times \frac{9}{8} = \frac{63}{32}$$

(b) $\frac{-2}{5}\times\left(\frac{3}{8}-25\right)=\frac{-2}{5}\times\left(\frac{3-200}{8}\right)=\frac{-2}{5}\times\frac{-197}{8}=\frac{197}{20}$

(c) $\frac{-3}{8}\times\left(\frac{4}{7}+\frac{-11}{7}\right)=\frac{-3}{8}\left(\frac{4-11}{7}\right)=\frac{-3}{8}\times\frac{-7}{7}=\frac{+21}{56}=\frac{3}{8}$

6. (a) $\frac{-23}{11}=\frac{11}{-23}$ (b) $\frac{15}{27}=\frac{27}{15}$ (c) $\frac{-4}{-7}=\frac{-7}{-4}$

(d) $\frac{4}{5}\times\frac{15}{18}=\frac{12}{18}$ (e) $-1\times\frac{-3}{10}\Rightarrow\frac{3}{10}$ (f) $\frac{-4}{5}\times\frac{-2}{7}=\frac{8}{35}$

$\frac{18}{12}\Rightarrow\frac{6}{4}=\frac{3}{2}$ Reciprocals $=\frac{10}{3}$ $\frac{35}{8}$ Reciprocals.

7. (a) $x(y-z)=x\times y-x\times z$

$$\frac{-3}{4}\left(\frac{2}{3}-\frac{-5}{6}\right)=\left(\frac{-3}{4}\times\frac{2}{3}\right)-\left(\frac{-3}{4}\times\frac{-5}{6}\right)$$

$$\frac{-3}{4}\left(\frac{4+5}{6}\right)=\frac{-1}{2}-\frac{5}{8}$$

$$\frac{-3}{4}\times\frac{9}{6}=\frac{-4-5}{8}$$

$$\frac{-9}{8}=\frac{-9}{8}$$ **Proved.**

(b) $x\times(y+z)=x\times y+x+z$

$$\frac{-3}{4}\left(\frac{2}{3}+\frac{-5}{6}\right)=\frac{-3}{4}\times\frac{2}{3}+\frac{-3}{4}\times\frac{-5}{6}$$

$$\frac{-3}{4}\left(\frac{4-5}{6}\right)=\frac{-1}{2}+\frac{5}{8}$$

$$\frac{-3}{4}\times\frac{-1}{6}=\frac{-4+5}{8}$$

$$\frac{1}{8}=\frac{1}{8}$$ **Proved.**

(c) $(x-y)^{-1}\neq x^{-1}+y^{-1}$

$$\left(\frac{-3}{4}-\frac{2}{3}\right)\neq\frac{4}{-3}+\frac{3}{2}$$

$$\left(\frac{-9-8}{12}\right)\neq\frac{-8+9}{6}\ \frac{-17}{2}\neq\frac{1}{6}$$ **Proved.**

(d) $|x^{-1}|=|x|^{-1}, |y^{-1}|=|y^{-1}|, |z|^{-1}=|z|^{-1}$

$$\left|\frac{4}{3}\right|=\left|-\frac{4}{3}\right| \quad\Rightarrow\quad \left|\frac{2}{3}\right|=\left|\frac{3}{2}\right|, \quad \left|\frac{6}{5}\right|=\left|\frac{6}{5}\right|$$

$$\frac{4}{3}=\frac{4}{3}, \quad\Rightarrow\quad \frac{2}{3}=\frac{2}{3}, \quad \frac{6}{5}=\frac{6}{5}$$ **Proved.**

8. (a) $(-36)\times\frac{-35}{76}\times\frac{19}{15}\times\left(\frac{3}{-2}\right)^{-1}$

$$-\overset{12}{36}\times\frac{\overset{7}{35}}{\underset{38}{76}}\times\frac{19}{\underset{3}{15}}\times\frac{-2}{3}$$

$$(3)=\frac{84}{6}=14$$

(b) $\left(\frac{-4}{7}\right)\times\left(\frac{-18}{11}\right)\times\left(\frac{44}{9}\right)\times(-14)$

$$\frac{-4}{7}\times\frac{\overset{2}{-18}}{11}\times\frac{\overset{4}{44}}{9}\times\overset{2}{-14}=-4\times-2\times4\times-2=-64$$

Exercise 1.5

1. (a) $\frac{5}{9}$ by 15 $\Rightarrow$ $\frac{5}{9}\div15=\frac{\overset{1}{5}}{9}\times\frac{1}{\underset{3}{15}}=\frac{1}{27}.$

(b) $\frac{7}{18}$ by $\frac{-15}{51}$ $\Rightarrow$ $\frac{7}{18}\div\frac{-14}{51}=\frac{7}{\underset{6}{18}}\times\frac{\overset{7}{51}}{\underset{2}{-14}}=\frac{-7}{12}.$

(c) $\frac{10}{33}$ by $\frac{2}{-11}$ $\Rightarrow$ $\frac{10}{33}\div\frac{2}{-11}=\frac{\overset{5}{10}}{\underset{3}{33}}\times\frac{\overset{1}{-11}}{2}=\frac{-5}{3}=\frac{-5}{3}.$

(d) -18 by $\frac{-36}{37}$ $\Rightarrow$ $-18\div\frac{-36}{37}=-\overset{1}{18}\times\frac{-37}{\underset{2}{36}}=\frac{-37}{2}.$

2. (a) $\frac{10}{21}\div\frac{8}{9}=\frac{8}{9}\div\frac{10}{21}$ $\Rightarrow$ $\frac{\overset{5}{10}}{\underset{7}{21}}\times\frac{\overset{3}{9}}{\underset{4}{8}}\neq\frac{\overset{4}{8}}{9}\times\frac{\overset{7}{21}}{\underset{5}{10}}$

$$\frac{15}{28}\neq\frac{28}{15}$$

Hence, it is false.

(b) $\frac{26}{15}\div\left(\frac{1}{5}+\frac{7}{3}\right)=\frac{26}{15}\div\frac{1}{5}+\frac{26}{15}\div\frac{7}{3}$

L.H.S. $\frac{26}{15}\div\left(\frac{3+35}{15}\right)$

R.H.S. $\frac{26}{15}\div\frac{38}{15}\neq\frac{26}{3}+\frac{26}{35}$

$$\frac{26}{\underset{1}{15}}\times\frac{\overset{1}{15}}{38}\neq\frac{910+78}{105}$$

$$\frac{26}{38}\neq\frac{988}{105}$$

Hence, it is false.

(c) $\left(\frac{9}{5}+\frac{4}{25}\right)\div\frac{-5}{7}=\frac{9}{5}\div\left(\frac{-5}{7}\right)+\frac{4}{25}\div\left(\frac{-5}{7}\right)$

$\left(\frac{9}{5}+\frac{4}{25}\right)\div\frac{-5}{7}=\left(\frac{9}{5}\div\frac{-5}{7}\right)+\left(\frac{4}{25}\div\frac{-5}{7}\right)$

L.H.S. $\left(\frac{45+4}{25}\right)\div\frac{-5}{7}$

$\frac{49}{25}\times\frac{7}{-5}$

$\Rightarrow \quad \frac{343}{-125}=\frac{-343}{125}$

R.H.S. $\frac{9}{5}\times\frac{7}{-5}+\frac{4}{25}\times\frac{7}{-5}$

$=\frac{9}{5}\times\frac{7}{5}+\frac{4}{25}\times\frac{7}{-5}$

$\Rightarrow \quad =\frac{-63}{25}+\frac{-28}{125}$

$\frac{-315+(-28)}{125}=\frac{-343}{125}$

$\frac{-343}{125}=\frac{-343}{125}$

L.H.S. = R.H.S. Hence, it is true.

(d) $\left(\frac{9}{5}-\frac{17}{40}\right)\div\frac{10}{3}=\left(\frac{9}{20}\div\frac{10}{3}\right)-\left(\frac{17}{40}\div\frac{10}{3}\right)$

L.H.S. $\left(\frac{9}{5}-\frac{17}{40}\right)\div\frac{10}{3}=\left(\frac{72-17}{40}\right)\times\frac{3}{10}=\frac{55}{40}\times\frac{3}{10}=\frac{165}{400}$

R.H.S. $\left(\frac{9}{20}\div\frac{10}{3}\right)-\left(\frac{17}{40}\div\frac{10}{3}\right)=\frac{9}{20}\times\frac{3}{10}-\frac{17}{40}\times\frac{3}{10}$

$\frac{27}{200}-\frac{51}{400} \quad \Rightarrow \quad \frac{54-51}{400}$

$\frac{165}{400}\neq\frac{3}{400}$

L.H.S. $\neq$ R.H.S. Hence, it is false.

(e) $\left(\frac{-3}{4}\div\frac{27}{15}\right)\div\frac{12}{-5}=\frac{-3}{4}\div\left(\frac{27}{15}\div\frac{12}{-5}\right)$

L.H.S. $=\left(\frac{-3}{4}\div\frac{27}{15}\right)\div\frac{12}{-5}$

$=\left(\frac{-3}{4}\times\frac{15}{27}\right)\div\frac{12}{-5}$

$=\frac{-45}{108}\times\frac{-5}{12}=\frac{75}{432}$

R.H.S. $= \frac{-3}{4} \div \left(\frac{27}{15} \div \frac{12}{-5}\right)$

$= \frac{-3}{4} \times \frac{15}{27} \times \frac{-5}{12} = \frac{75}{432}$

$\frac{75}{432} = \frac{75}{432}$

L.H.S. = R.H.S.

3. Fill in the blanks :

(a) $\frac{-35}{17} \div 1 = \frac{\mathbf{-35}}{\mathbf{17}}$ (b) $\frac{-25}{18} \div \frac{-25}{18} = \mathbf{1}$

(c) $\left(\frac{3}{5} - \frac{1}{2}\right) + \frac{\mathbf{3}}{\mathbf{4}} = \frac{3}{5} \div \frac{3}{4} - \frac{1}{2} \div \frac{3}{4}$ (d) $\frac{5}{6} \div \frac{2}{-3} = \frac{5}{6} \times \frac{\mathbf{-3}}{\mathbf{2}}$

4. (a) $\left(\frac{-3}{29} \div \frac{9}{87}\right) \div \frac{-1}{7} \Rightarrow \left(\frac{-3}{29} \times \frac{\overset{3}{87}}{9}\right) \div \frac{-1}{7} = \left(\frac{-9}{9}\right) \div \frac{-1}{7}$

$\Rightarrow \frac{-9}{9} \times \frac{7}{-1} = 7.$

(b) $\left(\frac{5}{9} \div \frac{15}{36}\right) \div \frac{5}{-6} = \left(\frac{5}{9} \times \frac{\overset{4}{36}}{\underset{3}{15}}\right) \div \frac{5}{-6} = \frac{4}{3} \times \frac{\overset{2}{-6}}{5} = \frac{-8}{5}.$

Exercise 1.6

1. (a) $x = \frac{1}{5}, y = \frac{1}{4}$

Required rational number $= \frac{1}{2}(x + y)$

$\frac{1}{2}\left(\frac{1}{5} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{4+5}{20}\right) = \frac{1}{2} \times \frac{9}{20} = \frac{9}{40}.$

(b) $x = \frac{-5}{6}, y = \frac{-2}{5}$

Rational number between $= \frac{1}{2}(x + y)$

$\Rightarrow \frac{1}{2}\left(\frac{-5}{6} + \frac{-2}{5}\right)\frac{1}{2}\left(\frac{-25+(-2)}{30}\right) = \frac{-37}{60}$

2. Let q_1, q_2 the two required rational numbers between $x = \frac{2}{3}$

$y = \frac{1}{2}$

$q = \frac{1}{2}(x + y) = \frac{1}{2}\left(\frac{-2}{3} + \frac{1}{4}\right)$

$= \frac{1}{2}\left(\frac{-8+3}{12}\right) = \frac{1}{2} \times \frac{-5}{12} = \frac{-5}{24}$

$$q_2 = \frac{1}{2}(q_1 + \text{smaller number})$$

$$= \frac{1}{2}\left(\frac{-5}{24} + \left(\frac{-2}{3}\right)\right)$$

$$= \frac{1}{2}\left(\frac{-5-36}{24}\right) = \frac{1}{2} \times \frac{-21}{24}$$

$$= \frac{-21}{48} = \frac{-5}{24}, \frac{21}{48}$$

3. Let q_1, q_2, q_3 the three rational number between $x = \frac{-11}{4}$, $y = \frac{6}{5}$

$$q_1 = \frac{1}{2}(x + y) = \frac{1}{2}\left(\frac{-11}{4} + \frac{6}{5}\right) = \left(\frac{-55+24}{20}\right) \times \frac{1}{2}$$

$$= \frac{-31}{20} \times \frac{1}{2} = \frac{-31}{40}$$

$$q_2 = \frac{1}{2}\left(\frac{-31}{40} + \frac{-11}{4}\right) = \frac{1}{2}\left(\frac{-31+(-110)}{40}\right) = \frac{-141}{80}$$

$$q_3 = \frac{1}{2}\left(\frac{-141}{80} + \frac{6}{5}\right) = \frac{1}{2}\left(\frac{-141+16}{80}\right) = \frac{-45}{160} = \frac{-9}{32}.$$

4. $q_1 = \frac{1}{2}\left(-1 + \frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-2-1}{2}\right) = \frac{-3}{4}$

$$q_2 = \frac{1}{2}\left(\frac{-3}{4} + \frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-3-2}{4}\right) = \frac{-5}{8}$$

$$q_3 = \frac{1}{2}\left(\frac{-5}{8} + \frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-5-4}{8}\right) = \frac{-9}{16}$$

$$q_4 = \frac{1}{2}\left(\frac{-9}{16} + \frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-9-8}{16}\right) = \frac{-17}{32}$$

5. Let q_1, q_2, q_3, q_4 and q_5 the rational number between $\frac{-4}{7}$ and $\frac{-3}{7}$.

$$q_1 = \frac{1}{2}\left(\frac{-4}{7} + \frac{3}{7}\right) \quad \Rightarrow \quad \frac{1}{2}\left(\frac{-4+3}{7}\right) = \frac{-1}{14}$$

$$q_2 = \frac{1}{2}\left(\frac{-1}{14} + \frac{-4}{7}\right) \quad \Rightarrow \quad \frac{1}{2} \times \left(\frac{-1-8}{14}\right) = \frac{-9}{28}$$

$$q_3 = \frac{1}{2}\left(\frac{-9}{28} + \frac{-4}{7}\right) \quad \Rightarrow \quad \frac{1}{2} \times \left(\frac{-9-16}{28}\right) = \frac{-25}{56}$$

$$q_4 = \frac{1}{2}\left(\frac{-25}{56} + \frac{-4}{7}\right) \quad \Rightarrow \quad \frac{1}{2} \times \left(\frac{-25-32}{56}\right) = \frac{-57}{112}$$

$$q_5 = \frac{1}{2}\left(\frac{-57}{112} + \frac{-4}{7}\right) \quad \Rightarrow \quad \frac{1}{2} \times \left(\frac{-57-64}{112}\right) = \frac{-121}{224}.$$

6. Let $q_1, q_2, q_3 .... q_6$ be the rational numbers between -1 and 0.

$q_1 = \frac{1}{2}(-1+0) = \frac{-1}{2}$ $\quad$ $q_2 = \frac{1}{2}\left(\frac{-1}{2}+0\right) = \frac{-1}{4}$

$q_3 = \frac{1}{2}\left(\frac{-1}{4}+0\right) = \frac{-1}{8}$ $\quad$ $q_4 = \frac{1}{2}\left(\frac{-1}{8}+0\right) = \frac{-1}{16}$

$q_5 = \frac{1}{2}\left(\frac{-1}{16}+0\right) = \frac{-1}{32}$ $\quad$ $q_6 = \frac{1}{2}\left(\frac{-1}{32}+0\right) = \frac{-1}{64}$.

7. Let q_1, q_2, q_3, q_4, q_5 be the rational numbers between $\frac{-1}{2}$ and $\frac{5}{4}$.

$q_1 = \frac{1}{2}\left(\frac{-1}{2}+\frac{5}{4}\right) = \frac{1}{2}\times\left(\frac{-2+5}{4}\right) = \frac{-3}{8}$

$q_3 = \frac{1}{2}\left(\frac{-3}{8}+\frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-3+(-4)}{8}\right) = \frac{-7}{16}$

$q_4 = \frac{1}{2}\left(\frac{-7}{16}-\frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-7(-8)}{16}\right) = \frac{-15}{32}$

$q_5 = \frac{1}{2}\left(\frac{-15}{32}+\frac{-1}{2}\right) = \frac{1}{2}\left(\frac{-15-16}{32}\right) = \frac{31}{64}$.

Exercise 1.7

1. $\because$ Quantity of water that required for one cup of tea $= \frac{3}{4}$ cup $= \frac{3}{4}\times 20$

$\therefore$ Quantity of water required for 20 cups of tea = 15 cup.

2. Length of piece of wood cut $= \frac{35}{4}$ m

Number of pieces of qual size = 4

$\therefore$ Length of each piece $= \frac{35}{4}\div\frac{4}{1} = \frac{35}{4}\times\frac{1}{4} = \frac{35}{16} = 2\frac{3}{16}$.

3. $\because$ Cost of 35/2 kg of potatoes = ₹ $\frac{665}{4}$

$\therefore$ Cost of 1 kg of potatoes = ₹ $\frac{665}{4}\div\frac{35}{2}$ $\frac{\overset{19}{\cancel{665}}}{\underset{2}{\cancel{4}}}\times\frac{2}{35} = \frac{19}{2}$ = ₹ $9\frac{1}{2}$.

4. $\because$ Cost of 1 metre cloth = ₹ $\frac{38}{7}$

$\because$ Cost of $\frac{7}{2}$ metre cloth $= \frac{38}{7}\times\frac{7}{2}$ = ₹ 19

$\because$ Cost of 1 metre ribbon = ₹ $\frac{55}{2}$

$\because$ Cost of $\frac{19}{4}$ metre ribbon = ₹ $\frac{55}{2}\times\frac{19}{4}$ = ₹ $\frac{1045}{8}$

She spent = ₹ $\left(19+\frac{1045}{8}\right) = \left(\frac{152+1045}{8}\right)$ = ₹ $\left(\frac{1197}{8}\right)$ = ₹ $149\frac{5}{8}$.

5. ∵ Distance covered by a bus in 1 hour $= \frac{404}{5}$ km $= \frac{19}{4}$ hour = ?

∴ Bus will cover the distance in $\frac{19}{4}$ hours $= \left(\frac{404}{5} \times \frac{19}{4}\right)$ km.

$$= \frac{\overset{101}{404}}{5} \times \frac{19}{4} = \frac{1919}{5} \text{ km.}$$

6. Total number of flowers in his shop = 400

Number of flowers sold in $= \frac{1}{10}$th of 400

$$= 400 \times \frac{1}{10} = 40 \text{ flower}$$

Remaining flowers $= 400 - 40 = 360$ flower

Number of flowers sold in the evening $= 360 \times \frac{1}{3} = 120$ flower

Remaining flowers $= 360 - 120 = 240$ flower

Number of flowers that with her $= 240 \times \frac{1}{6} = 40$ flower

then remaining flowers $= 240 - 40 = 200$ flower

total no. of flowers that sold $= 400 + 120 = 160\,160$ flower.

No. of flowers left $= 400 - 160 = 240$ flowers.

7. Mrs. Khanna now weighs = 95 kg

Suppose mrs. Khanna weighs at the start = x kg

then, according to question $\left(x - \frac{11}{2} + \frac{9}{4} - \frac{15}{4}\right) = 95 \text{ kg}$

$$\left(\frac{4x - 22 + 9 - 15}{4}\right) = 95 \text{ kg}$$

$$\left(\frac{4x - 28}{4}\right) \text{kg} \times \frac{95}{1} \text{ kg}$$

$4x - 28 = 380$ (cross multiplication)

$4x = 380 + 28$

$4x = 408$

$x = \frac{408}{4} = 102 \text{ kg.}$

Hence, the weight of Mrs. Khanna was 102 at the begining.

MCQ's

1.(c) **2.** (c) **3.** (b) **4.** (d) **5.** (b) **6.** (d) **7.** (c) **8.** (c) **9.** (b) **10.** (a) **11.** (b) **12.** (b) **13.** (d) **14.** (c).

2. Powers

Exercise 2.1

1. (a) $(5)^3$ ⇒ Base = 5, exponents = 3

(b) $(2)^{-5}$ ⇒ Base = 2, exponents = – 5

(c) $(-7)^4$ ⇒ Base = – 7, exponents = 4

(d) $(3)^5$ $\Rightarrow$ Base = 3, exponents = 5

(e) $\left(\frac{-2}{5}\right)^3$ $\Rightarrow$ Base = $\frac{-2}{5}$, exponents = 3

(f) $\left(\frac{4}{5}\right)^{-2}$ $\Rightarrow$ Base = $\frac{4}{5}$, exponents = – 2

2. (a) $4^{\frac{3}{2}} = (2\times 2)^{3/2} = (2)^{2\times 3/2} = (2)^3 = 8$

(b) $8^{2/3} = (2)^{3\times\frac{2}{3}} = (2)^2 = 4$

(c) $(343)^{2/3} = (7)^{3\times\frac{2}{3}} = (7)^2 = 49$

(d) $(32768)^{1/15} = (2)^{15\times\frac{1}{15}} = 2$

(e) $(279936)^{1/7} = (6)^{7\times\frac{1}{7}} = 6$

(f) $(343)^{-1/3} = \left(\frac{1}{343}\right)^{1/3} = \frac{(1)^{1./3}}{(7)^{3\,\frac{1}{3}}} = \frac{1}{7}$

3. (a) $\left(\frac{32}{243}\right)^{4/5} = \left(\frac{(2)^5}{(3)^5}\right)^{4/5} \Rightarrow \frac{(2)^{5\times 4/5}}{(3)^{5\times 4/5}} \Rightarrow \frac{32}{81} \Rightarrow \frac{32}{81}$

(b) $\left(\frac{25}{49}\right)^{7/2} = \left(\frac{(5)^2}{(7)^2}\right)^{7/2} \Rightarrow \frac{(5)^{2\times 7/2}}{(7)^{2\times 7/2}} \Rightarrow \frac{(5)^7}{(7)^7} \Rightarrow \frac{78125}{823543}$

(c) $\left(\frac{1}{9}\right)^{-1/2} \Rightarrow \left(\frac{(5)^4}{(3)^4}\right)^{-1/4} \Rightarrow \frac{3}{5}$

(d) $\left(\frac{1}{9}\right)^{-1/2} \Rightarrow (9)^{1/2} \Rightarrow 3$

4. (a) $(0.04)^{5/2} = (0.2)^{5/2} = (0.000032)$

(b) $(0.000729)^{5/6} = \{(0.027)^2\} = (0.027)^{2\times\frac{5}{6_3}} = (0.027)^{5/3}$

(c) $(0.125)^{2/3} = \{(0.5)^3\}^{2/3} = (0.5)^{3\times\frac{2}{3}} = 0.25$

(d) $(0.00064)^{5/6} = \{(0.2)^6\}^{5/6} = (0.2))^{6\times 5/6} = 0.000032$

5. (a) $(27)^{2/3} \times (27)^{1/3} \times (27)^{-4/3}$

$= [(3)^3]^{2/3} \times [(3)^3]^{1/3} \times [(3)^3]^{-4/3}$

$= (3)^{3\times 2/3} \times (3)^{3\times 1/3} \times (3)^{3\times -4/3}$

$= (3)^2 \times (3)^1 \times (3)^{-4}$

$= (3)^{2+1-4} = (3)^{-1} = \frac{1}{3}$

(b) $\left(\frac{27}{125}\right)^{\frac{-2}{3}} \times \left(\frac{27}{125}\right)^{\frac{-4}{3}} = \left(\frac{125}{27}\right)^{2/3} \times \left(\frac{125}{27}\right)^{4/3}$

$= \frac{(5)^{3\times\frac{2}{3}}}{(3)^{3\times\frac{2}{3}}} \times \frac{(5)^{3\times\frac{4}{3}}}{(3)^{3\times\frac{4}{3}}} = \frac{(5)^2}{(3)^2} \times \frac{(5)^4}{(3)^4}$

$= \left(\frac{5}{3}\right)^2 \times \left(\frac{5}{3}\right)^4$

$= \left(\frac{5}{3}\right)^6 = \frac{15625}{729}$

(c) $\left[(729)^{\frac{-5}{3}}\right]^{-1/2} = \left[\left(\frac{1}{729}\right)^{\frac{5}{3}}\right]^{-1/2} = \left(\frac{1}{(9)^{3\times 5/3}}\right)^{-1/2} = (3)^{2\times 5\times\frac{1}{2}} = 243$

(d) $\left(\frac{2}{3}\right)^{4/3}\left(\frac{2}{13}\right)^{5/3} = \left(\frac{2}{13}\right)^{\frac{4}{3}+\frac{5}{3}} = \left(\frac{2}{13}\right)^{\frac{9^3}{3}} = \left(\frac{2}{13}\right)^3$

6. (a) $\{(3)^2 + (4)^2\}^{1/2} = (3+4) = (9+6)^{1/2} = (25)^{1/2} = 5$

(b) $\{(5)^2 + (12)^2\}^{3/2} = (25+144)^{3/2} = \{(169)\}^{3/2} = (13)^{2\times\frac{3}{2}} = 2197$

(c) $(17^2 - 8^2) = (289-64)^{1/2} = (225)^{1/2} = (15)^{2\times\frac{1}{2}} = 15$

(d) $(1^3 + 2^3 + 3^3 + 4^3)^{3/2} = (1+8+27+64)$

$= (100)^{\frac{-3}{2}} = (10)^{3\times\frac{-3}{2}}$

$= 10^{-3} = \frac{1}{(1000)}$

Exercise-2.2

1. (a) $5^x = 625 \quad \Rightarrow \quad (5)^x = (5)^4 \quad \Rightarrow \quad x = 4$

(b) $3^{4x} = \frac{1}{81} \quad \Rightarrow \quad 3^{4x} = \left(\frac{1}{(3)^4}\right) \quad \Rightarrow \quad 3^{4x} = \frac{1}{(3)^4}$

$\Rightarrow \quad (3)^{4x} = (3)^{-4} \quad \Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = \frac{-4}{4} = -1$

(c) $4^x = \frac{1}{16} \quad \Rightarrow \quad 4^x = \frac{1}{(4)^2}$

$\Rightarrow \quad 4^x = (4)^{-2} \quad \Rightarrow \quad x = -2$

(d) $7^{2x-1} = 343 \quad \Rightarrow \quad 7^{2x-1} = (7)^4$

$2x - 1 = 4 \quad \Rightarrow \quad 2x = 4 + 1$

$\Rightarrow \quad x = \frac{5}{2}$

(e) $2^x = (\sqrt{2})^4 \times (128)^{1/7} \quad \Rightarrow \quad ((2)^{1/2})^4 \times (128)^{1/7}$

$2^x = (2)^{\frac{1}{x} \times 4^2} \times (2)^{7 \times \frac{1}{2}} \quad \Rightarrow \quad (2)^2 \times (2)^1$

$(2)^x = (2)^3 \quad \Rightarrow \quad x = 3$

(f) $4^x \times 2^x = (32)^{1/5} \times 8^{1/3} \quad \Rightarrow \quad 2^{2x} \times 2^x = 2^{5 \times 1/5} \times 2^{3 \times 1/3}$

$2^{2x+x} = 2^2 \quad \Rightarrow \quad 2x + x = 2$

$3x = 2 \quad \Rightarrow \quad x = \frac{2}{3}$

2. (a) $\sqrt{5} = 5^{1/2}$ (b) $4\sqrt{7} = 7^{1/4}$

(c) $7\sqrt{1280} = (1280)^{1/7}$ (d) $7\sqrt{\frac{32}{79}} = \left(\frac{32}{79}\right)^8$

3. (a) $(13)^{1/2} = \sqrt{13}$ (b) $(112)^{1/5} = \sqrt{112}$

(c) $\left(\frac{7}{12}\right)^{1/9} = 9\sqrt{7/12}$ (d) $\left(\frac{516}{63}\right)^{-1/14} = \left(\frac{63}{516}\right)^{14} = 14\sqrt{\frac{63}{516}}$

4. (a) $2\sqrt{6} = \sqrt{4 \times 6} = \sqrt{24}$ (b) $7\sqrt{6} = \sqrt{49 \times 6} = \sqrt{294}$

(c) $10\sqrt{5} = \sqrt{100 \times 5} = \sqrt{500}$ (d) $\frac{2}{3}\sqrt{40} = \sqrt{\frac{4}{9} \times 40} = \sqrt{\frac{1600}{9}}$

5. (a) $\sqrt{108} = \underline{3 \times 3} \times 3 \times \underline{2 \times 2} \times \underline{2 \times 2} = 3 \times 2\sqrt{3} = 6\sqrt{3}$

(b) $\sqrt{99} = \sqrt{3 \times 3 \times 11} = 3\sqrt{11}$

(c) $\sqrt{405} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times 5} = 9\sqrt{5}$

(d) $\sqrt{162} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times 2} = 9\sqrt{2}$

6. (a) $\sqrt{63} + \sqrt{28} \times \sqrt{112}$

$= \sqrt{\underline{3 \times 3} \times 7} + \sqrt{\underline{2 \times 2} \times 7} + \sqrt{\underline{2 \times 2} \times 2 \times \underline{2 \times 2} \times 7}$

$= 3\sqrt{7} + 2\sqrt{7} + 4\sqrt{7}$

$= (3 + 2 + 4)\sqrt{7} = 9\sqrt{7}$

(b) $\sqrt{45} + \sqrt{8} - \sqrt{80} + \sqrt{18} + \sqrt{5} - \sqrt{2}$

$= \sqrt{3 \times 3 \times 5} + \sqrt{2 \times 2 \times 2} - \sqrt{2 \times 2 \times 2 \times 2 \times 5} + \sqrt{3 \times 3 \times 2} + \sqrt{5} - \sqrt{2}$

$= 3\sqrt{5} + 2\sqrt{2} - 4\sqrt{5} + 3\sqrt{2} + \sqrt{5} - \sqrt{2}$

$= 4\sqrt{5} - 4\sqrt{5} + 2\sqrt{2} + 3\sqrt{2} - \sqrt{2}$

$= \sqrt{2}\,(2 + 3 - 1)$

$= \sqrt{2}(4) = 4\sqrt{2}$

7. (a) $\frac{7}{\sqrt{5}} = \frac{7 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{7\sqrt{5}}{5}$

(b) $\frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{3 \times 3} = \frac{2\sqrt{3}}{9}$

(c) $\frac{5}{\sqrt{27}} = \frac{5 \times \sqrt{27}}{\sqrt{27} \times \sqrt{27}} = \frac{5\sqrt{27}}{27} = 5 \times 3 \times = \frac{5\sqrt{3}}{9}$

(d) $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{\sqrt{10}}{\sqrt{25}} = \frac{\sqrt{10}}{5}$ Ans.

(e) $\frac{6}{4-\sqrt{10}} = \frac{6}{4-\sqrt{10}} \times \frac{4+\sqrt{10}}{4+\sqrt{10}}$

$= \frac{6(4+\sqrt{10})}{(4)^2 - (\sqrt{10})^2}$

$= \frac{6(4+\sqrt{10})}{16-10} = \frac{6(4+\sqrt{10})}{6}$

$= 4+\sqrt{10}$

(f) $\frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5})^2 - (\sqrt{3})^2}$

$\Rightarrow \quad = \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$

MCQ

1. (b) **2.** (c) **3.** (c) **4.** (a) **5.** (d) **6.** (d) **7.** (c) **8.** (d)

3. Square and Square Root

Exercise 3.1

1. (a) $12 = (12)^2 \quad \Rightarrow \quad 12\times 12 = 144$

(b) $21 = (21)^2 \quad \Rightarrow \quad 21\times 21 = 441$

(c) $44 = (44)^2 \quad \Rightarrow \quad 44\times 44 = 1936$

(d) $65 = (65)^2 \quad \Rightarrow \quad 65\times 65 = 4225$

(e) $102 = (102)^2 \quad \Rightarrow \quad 102\times 102 = 10404$

2. (a) $625 = \underline{5\times 5} \times \underline{5\times 5}$

$\sqrt{625} = 5\times 5 = 25$

5	625
5	125
5	25
5	5
	1

(b) $729 = \underline{3\times 3} \times \underline{3\times 3} \times \underline{3\times 3}$

$\sqrt{729} = 3\times 3\times 3$

$= 27$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(c) $2304 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{2\times 2}\times \underline{3\times 3}$

$\sqrt{2304} = 2\times 2\times 2\times 2\times 3$

$= 48$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(d) $9801 = \underline{3\times 3}\times \underline{3\times 3}\times \underline{11\times 11}$

$\sqrt{9801} = 3\times 3\times 11$

$= 99$

3	9801
3	3267
3	1089
3	363
11	121
11	11
	1

(e) $10000 = \underline{2\times 2}\times \underline{2\times 2}\times \underline{5\times 5}\times \underline{5\times 5}$

$\sqrt{10000} = 2\times 2\times 5\times 5$

$= 100$

5	10000
5	2000
5	400
5	80
2	16
2	8
2	4
2	2
	1

(f) $100489 = \underline{317\times 317}$

$\sqrt{100489} = 317$

317	100489
317	317
	1

(g) $5909761 = \underline{17 \times 17} \times \underline{13 \times 13} \times \underline{11 \times 11}$

$\sqrt{5909761} = 17 \times 13 \times 11$

$= 2431$

17	5909761
17	347633
13	20449
13	1573
11	121
11	11
	1

3. (a) $\frac{81}{169} = \left(\frac{9}{13}\right)^2$ then $\sqrt{\frac{81}{169}} = \sqrt{\left(\frac{9}{13}\right)^2} = \frac{9}{13}$

(b) $\frac{484}{9216} = \left(\frac{22}{96}\right)^2$ then $\sqrt{\frac{484}{9216}} = \sqrt{\left(\frac{22}{96}\right)^2} = \frac{22}{96}$

(c) $\frac{625}{1296} = \left(\frac{25}{36}\right)^2$

then $\sqrt{\frac{625}{1296}} = \sqrt{\left(\frac{25}{36}\right)^2} = \frac{25}{36}$

5	625
5	125
5	25
5	5
	1

5×5

6	1296
6	216
6	36
6	6
	1

6×6

(d) $0.04 = (0.2)^2$ thus $\sqrt{0.4} = \sqrt{(0.2)^2} = 0.2$

(e) $0.0009 = (0.03)^2$ thus $\sqrt{0.0009} = \sqrt{(0.03)^2} = 0.03$

(f) $0.0625 = (0.25)^2$ thus $\sqrt{0.0625} = 0.25$

(g) $4\frac{29}{49} = \frac{225}{49} = \left(\frac{15}{7}\right)^2$ thus $\sqrt{\frac{225}{49}} = \sqrt{\left(\frac{15^2}{7}\right)} = \frac{15}{7}$

(h) $38\frac{11}{25} = \frac{961}{25} = \left(\frac{31}{5}\right)^2$ thus $\sqrt{\frac{961}{25}} = \frac{31}{5}$

(i) $23\frac{26}{121} = \frac{2809}{121} = \left(\frac{53}{11}\right)^2$ thus $\sqrt{\frac{2809}{121}} = \frac{53}{11}$

(j) $101\frac{1}{400} = \frac{40401}{400} = \left(\frac{201}{20}\right)^2$ thus $\sqrt{\frac{40401}{400}} = \frac{201}{20}$

(k) $3\frac{334}{3025} = \frac{9409}{3025} = \left(\frac{97}{55}\right)^2$ thus $\sqrt{\frac{9409}{3025}} = \frac{97}{55}$

(l) $63.6804 = \frac{636804}{10000} = \left(\frac{798}{100}\right)^2$ thus $\sqrt{\frac{636804}{10000}} = \frac{798}{100}$

4. Area of square field = 31684 m^2

We know that, Area of square = side × side

then, $31684 = (\text{side})^2$

or $\text{side} = \sqrt{31684}$

then $\text{side} = \sqrt{31684}$

$= \sqrt{(178)^2}$

$= 178\,m$

Length of each sides is 178 m.

2	31684
2	15842
89	7921
89	89
	1

5. Let the number of rows = x

$\therefore$ No. of plants in each row = x

$\therefore$ Total number of plants = $x \times x = x^2$

According to the question,

$x^2 = 784$

or $x = \sqrt{784}$

$= \sqrt{\underline{2\times 2}\times \underline{2\times 2}\times \underline{7\times 7}}$

$= 2\times 2\times 7 = 28$

So, the numbers of rows = 28

and the number of plants in each row = 28.

2	784
2	392
2	196
2	98
7	49
7	7
	1

6. By prime factorization

We have $\overline{5\times 3}\times \overline{7\times 7}$

To make 735 a perfect square,

it should be multiply by 15

so that it become a perfect square.

If we multiply 615 the number so obtained

$735\times 15 = 11025$

$\underline{3\times 3}\times \underline{5\times 5}\times \underline{7\times 7} = 11025$

$3\times 5\times 7 = 105$

$= \sqrt{11025}$

thus square root is 105.

If we divide 735 by 15

the quotient 49 is also a perfect square.

5	735
3	147
7	49
7	7
	1

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

7. Find smallest number by which $3^6 \times 3^5$ must be multiplied or dividend so that the product (or quotient) become a perfect square.

$3^6 \times 3^5 = 3^{6+5} = 3^{11}$

$\underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}\times 3$

to make 36×35 a perfect square it should be multiply by 3 so that it become a perfect square.

If we multiply by 3 the number so obtained

$3^6 \times 3^5 \times 3 = \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3}\times \underline{3\times 3} = 531441$

$729 = \sqrt{531441}$

Thus square root is 729.

If we divide 6×35 by 3 the quotient 59049 is also a perfect square.

8. The last number which will be divisible by 8, 9 and 10 is the L.C.M. of 8, 9 and $10 = 2 \times 4 \times 9 \times 5 = 360$
On factorisin 360, we get

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

Therefore,to make perfect square it need to be multiplied by (2×5) i.e. 10.
Hence, required number $= 360 \times 10 = 3600$.

2	8, 9, 10
2	4, 9, 5
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5

Exercise 3.2

1. (a) $(31)^2 = 961$ 9 6 1 — Unit digit of the square is 1.

(b) $(122)^2 = 14884$ 1 4 8 8 4 — Unit digit of 14884 is 4.

(c) $(389)^2 = 1513211$ 5 1 3 2 — Unit digit is 1.

(d) $(3253)^2 = 10582009$ — Unit digit of square is 9.

(e) $(464)^2$ 2 1 5 2 9 6 — Unit digit is 6.

(f) $(63487)^2$ 4 0 3 0 5 9 9 1 6 9 — Unit digit is 9.

(g) $(12398)^2 = 153710404$ — Unit digit is 4.

2. We know that if a number ending with even number of zeros, the number must be a square number.
and if a number ending with odd number of zeros, the number is never a perfect square.
So, 20 number of square = 1 (odd)
So, it is not a perfect square.
900 number of zeros = 2 (even)
So, it is a perfect square.
10000 Number of zeros = 4 (even)
So it is a perfect square
400000 Number of zeros = 5 (odd)
So, it is not a perfect square.
16000 Number of zeros = 3 (odd)
So, it is not a perfect square.

3. (a) 25
$\because$ It is an odd number so, its square must be odd.
$\therefore$ $(25)^2 = 625$

(b) 84
$\because$ It is an even number, so it's square must be even.
$\therefore$ $(84)^2 = 7056$

(c) 107 = odd number
So, It is square must be odd.
$\therefore$ $(107) = 11449$

(d) 252 = even, so its square number even.
$\therefore$ $(252)^2 = 63504$

(e) $(459)^2$ = even number, so its square must be even.
$\therefore$ $(459)^2 = 210681$

4. Square numbers from 1 to 400 are as follows :

$(1)^2 = 1$, $(2)^2 = 4$, $(3)^2 = 9$, $(4)^2 = 16$, $(5)^2 = 25$,
$(6)^2 = 36$, $(7)^2 = 49$, $(8)^2 = 64$, $(9)^2 = 81$, $(10)^2 = 100$
$(11)^2 = 121$, $(12)^2 = 144$, $(13)^2 = 169$, $(14)^2 = 196$, $(15)^2 = 225$,
$(16)^2 = 256$, $(17)^2 = 289$, $(18)^2 = 324$, $(19)^2 = 361$, $(20)^2 = 400$.

5. (a) $1 + 3 + 5$ $(3)^2 = 9$

(b) $1 + 3 + 5 + 7$ $(4)^2 = 16$

(c) $1 + 3 + 5 + 7 + 8$ $(5)^2 = 25$

(d) $1 + 3 + 5 + 7 + 9 + 11$ $(6)^2 = 36$

(e) $1 + 3 + 5 + 7 + 9 + 11 + 13$ $(7)^2 = 49$

(f) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$ $(8)^2 = 64$

(g) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$ $(9)^2 = 81$

6. (a) $(15)^2 - (14)^2 = (15 + 14)(15 - 14)$ $(\because a^2 - b^2 = (a + b)(a - b)$

$(29)(1) = 29$

(b) $(77)^2 - (76)^2 = (77 + 76)(77 - 76) = (153)(1) = 153$

(c) $(524)^2 - (523)^2 = (524 + 523)(524 - 523) = (1047)(1) = 1047$

(d) $(1000)^2 - (999)^2 = (1000 + 999)(1000 - 999) = (1999)(1) = 1999$

(e) $11500^2 - (1499)^2 = (1500 + 1499)(1500 - 1499) = (2999)(1) = 2999$

7. (i) $m = 4$ then

$2m = 2 \times 4 = 8$

$m^2 - 1 = (4)^2 - 1 = 16 - 1 = 15$

$m^2 + 1 = (4)^2 + 1 = 16 + 1 = 17$

8, 15, 17

(ii) $m = 6$ then

$2m = 2 \times 6 = 12$

$m^2 - 1 = (6)^2 - 1 = 35$

$m^2 + 1 = (6)^2 + 1 = 37$

12, 35, 37

8. $7^2 = 49$

$67^2 = 4489$

$667^2 = 44889$

$6667^2 = 444889$

$66667^2 = 4444889$

Exercise 3.3

1. (a) 9801

	99
9	$\overline{98}$ $\overline{01}$
+ 9	81
	17 01
189	– 17 01
	0

= 99.

(b) 4624

	68
6	$\overline{46}$ $\overline{24}$
+ 6	– 36
128	10 24
	– 10 24
	0

= 68

(c) 7744

	88
8	$\overline{77}\ \overline{44}$
+ 8	– 64
168	1344
	– 1344
	0

= 88.

(d) 11025

	105
1	$\overline{11}\ \overline{0}\ \overline{25}$
+ 1	– 1
205	1025
	– 1025
	0

= 105

(e) 97344

	312
3	$\overline{9}\,\overline{73}\ \overline{44}$
+ 3	– 9
61	73
+ 1	– 61
622	1244
	– 1244
	0

= 312.

(f) 974169

	987
9	$\overline{97}\ \overline{41}\ \overline{69}$
+ 9	– 81
188	1641
+ 8	– 1504
1967	13769
	– 13769
	0

= 987.

(g) 1745041

	1321
1	$17\ \overline{4}\,5\,\overline{0}\,\overline{41}$
+ 1	–1
23	74
3	– 69
262	550
2	– 524
2641	2641
	– 2641
	0

= 1321.

(h) 4008004

	2002
2	$\overline{4}\ \overline{00}\,\overline{80}\,\overline{04}$
+ 2	– 4
40	00
+ 0	00
4002	8004
	– 8004
	0

= 2002.

2. (a) 2361

This shows that $(48)^2$ is less than 2361 by 57.

So 57 must be subtracted from 2361.

The perfect square no. = 2361 – 57 = 2304

$\sqrt{2304} = 48$

	48
4	$\overline{23}\ \overline{61}$
+ 4	–16
88	761
	–704
	57

	48
4	$\overline{23}\ \overline{04}$
+ 4	–16
88	704
	–704
	0

Hence, the required number is 57.

(b) 645

This shows that $(25)^2$ is less than 645 by 20.

So 20 must be subtracted from 645

Perfect square numbers = 645 – 20 = 625

$\sqrt{625} = 25$

	25
2	$\bar{6}\ \overline{45}$
+ 2	–4
45	245
	–225
	0

	25
2	$\bar{6}\ \overline{25}$
+ 2	–4
45	225
	–225
	0

Hence, the required number is 20.

(c) 1525

This shows that $(39)^2$ is less than 1525 by 4, so, 4 must be subtracted thus the required

1525 – 4 = 1521

$\sqrt{1521} = 39$

	39
3	$\overline{15}\ \overline{25}$
+ 3	–9
69	625
	–621
	4

	39
3	$\overline{15}\ \overline{21}$
+ 3	– 9
69	6 21
	– 621
	0

Hence, the required number is 4.

(d) 194491

This shows that $(441)^2$ is less than 194491 by 10.

So 10 must be subtracted thus required

$194491 - 10 = 194481$

$\sqrt{194481} = 441$

	441
4	$\overline{19}\ \overline{44}\ \overline{91}$
+ 4	–16
84	344
4	–336
881	891
	–881
	10

	441
4	$\overline{19}\ \overline{44}\ \overline{81}$
	–16
84	344
	–336
881	891
	–881
	0

(e) 273682

This shows that $(523)^2$ is less than 273682 by 153. So 153 must be subtracted from 273682.

Square no. = $273682 - 153 = 273529$

$\sqrt{273529} = 523$

	523
5	$\overline{27}\ \overline{36}\ \overline{82}$
+ 5	–25
102	236
+ 2	–204
1043	3282
	–3129
	153

	523
5	$\overline{27}\ \overline{35}\ \overline{29}$
	–25
102	235
	–204
1043	3129
	–3129
	0

Hence, required number is 153.

3. (a) Let us find the square root of 5678.

Since 5678 is more than 75^2

the next square number will be 76^2

clearly $75^2 < 5678 < 76^2$

As $76 \times 76 = 5776$

required number = $5776 - 5678 = 98$

obtain 5776, 98 added to 5678.

The required number is 98.

	75
7	$\overline{56}\ \overline{78}$
+ 7	–49
145	778
	–725
	53

(b) Let us find the root of 9991.

Since 9991 is more than $(99)^2$

the next square number will be $(100)^2$

$$(99)^2 < 9991 < (100)^2$$

$$(100)^2 = 100 \times 100 = 10000$$

Required number $= 10000 - 9991$

$= 9$

So, to obtain 10000, 9 must be added in 9991.

Hence, required number is 9.

	99
9	$\overline{99}\ \overline{91}$
+ 9	–81
189	1891
	–1701
	190

(c) 4215

Let us find the root of 4215

Since, 4215 is more than $(64)^2$

the next square number will be $(65)^2$.

$\because$ $(64)^2 < 4215 < (65)^2$

$(65)^2 = 4225$

Required number $= 4225 - 4215 = 10$

	64
6	$\overline{42}\ \overline{15}$
+ 6	–36
124	615
	– 496
	119

	65
6	$\overline{42}\ \overline{25}$
+ 6	–36
125	6 25
	– 6 25
	0

To obtain 4225, 10 must be added to 4215.

Hence, the required number is 10.

(d) 306452

Let us find the root of 306452

Since 306452 is more than $(553)^2$

the next square number will be $(554)^2$.

$$(553)^2 < 306452 < (554)^2$$

$$(554)^2 = 306916$$

required number $= 306916 - 306452$

$= 464$

So, to obtain 306916, 464 must be added to 306452.

Hence, the required number is 464.

	553
5	$\overline{30}\ \overline{64}\ \overline{52}$
+ 5	–25
105	564
	–525
1103	3952
	–3309
	643

(e) 92700

Let us find the square root of 92700.

Since 92700 is more than $(304)^2$,

the next square number will be $(305)^2$

$305 = 305 \times 305 = 93025$

	304
3	$\overline{9}\ \overline{27}\ \overline{00}$
3	–9
604	2700
4	–2416
	284

	305
3	$\overline{9}\ \overline{30}\ \overline{25}$
	–9
605	3025
	–3025
	0

Required number = 93025 – 92700 = 325
to obtain 93025, 325 must be added to 92700.

$\sqrt{93025} = 305$

Hence, the required number is 325.

4. Greatest 4 digit number = 9999
Now we need to find the least number
which subtracted from 9999 given a perfect square
Required number = 9999 – 198 = 9801
$\because$ $\sqrt{9801} = 99$

	99
9	$\overline{99}\ \overline{99}$
9	–81
189	1899
	–1701
	198

5. The least number of 6 digits = 100000
We observe that

$$(316)^2 < 100000 < (317)^2$$

the number to be added $= (317)^2 - 100000$
$= 100489 - 100000 = 489$
Requred perfect square $= 100000 + 489$
$= 100489$

$\sqrt{100489} = 317$

Hence, the required number is 100489.

	316
3	$\overline{10}\ \overline{00}\ \overline{00}$
	–9
61	100
+ 2	–61
636	3900
	– 3816
	84

6. Area of a square field = 8649 m^2

$(side)^2 = 8649\ m = 93\ m$

Perimeter of square = 4 × side
$= 4 \times 93 = 372$

Now, suppose length of rectangle = $2x$
$\therefore$ breadth of rectangle = x
the perimeter of rectangle = perimeter of squares (Given)

$2(2x + x) = 372\ m$
$4x + 2x = 372\ m$

$$x = \frac{372}{6} = 62\ m$$

	93
9	$\overline{86}\ \overline{49}$
+ 9	–81
183	349
	–349
	0

length of rectangle = $62 \times 2\,\text{m} = 124\,\text{m}$
and breadth of rectangle = 62 m
then area of rectangle = $124\,\text{m} \times 62\,\text{m} = 7688\,\text{m}^2$.

7. First we need to find root of 9811.
This shows that $(99)^2$ is less than 9811 by 10.
So, the number of men left = 10
then, the number of man are arranged in rows
$= 9811 - 10 = 9801$
Number of men in each row = x
and number of row = x
Then, $x \times x = 9801$
$x = \sqrt{9801} = \sqrt{99 \times 99}$

	99
9	9811
+ 9	–81
189	1711
	–1701
	10

So, number of men in each row = 99

Exercise 3.4

1. (a) 84.64

	9.2
9	$\overline{84}\,.\,\overline{64}$
+ 9	–81
182	364
	–364
	0

$\sqrt{84.64} = 9.2$

(b) 180.0964

	13.42
1	$\overline{1}\,\overline{80}\,.\,\overline{09}\,\overline{64}$
+ 1	–1
23	80
+ 3	–69
264	1109
+ 4	–1056
2682	5364
	–5364
	0

$\sqrt{180.0964} = 13.42$

(c) 1785.0625

	42.25
4	$\overline{17}\,\overline{85}\,.\,\overline{06}\,\overline{25}$
+ 4	–16
82	185
+ 2	–164
842	2106
+ 2	–1684
8445	42225
	–42225
	0

$\sqrt{1785.0625} = 42.25$

(d) 0.104976

	0.324
3	$0.\,\overline{10}\,\overline{49}\,\overline{76}$
+ 3	–9
62	149
+ 2	–124
644	2576
	–2576
	0

$\sqrt{0.104976} = 0.324$

(e) 0.004489

	.067
0	0.004489
+ 0	00
06	44
+ 6	36
	889
127	889
	0

$\sqrt{0.004489} = 0.067$

(f) 14762.25

	121.5
1	$\bar{1}\ \overline{47}\ \overline{62}\ .\ \overline{25}$
+ 1	–1
22	47
+ 2	–44
241	362
+ 1	–241
2425	12125
	–12125
	0

$\sqrt{14762.25} = 121.5$

2. (a) 6.24

	2.497
2	$\bar{6}\ .\ \overline{24}\ \overline{00}\ \overline{00}$
+ 2	– 4
44	224
+ 4	– 176
489	4800
+ 9	– 4401
4987	39900
+ 7	– 34909
49949	499100
	– 449541
	49559

$\sqrt{6.24} = 2.49$

(b) 24.8

	4.979
4	$\overline{24}\ .\ \bar{8}$
+ 4	–16
89	880
+ 9	–801
987	7900
+ 7	–6909
949	99100
+ 9	–89541
	9559

$\sqrt{24.8} = 4.979$

(c) 625.5956

	25.0119
2	$\overline{625}$. $\overline{59}$ $\overline{56}$ $\overline{00}$ $\overline{00}$
+ 1	– 4
45	225
+ 5	–225
500	059
+ 0	–00
5001	5956
+ 1	–5001
50021	95500
+ 1	–50021
500229	4547900
	–4502061
	45839

$\therefore \quad \sqrt{625.5956} = 25.012$ (aprox)

(d) 15525.28

	124.600
1	$\bar{1}$ $\overline{55}$ $\overline{25}$. $\overline{28}$
+ 1	–1
22	55
+ 2	– 44
244	1125
+ 4	–976
2486	14928
+ 6	– 14916
	12000

$\therefore \quad \sqrt{15525.28} = 124.600$

3. (a) 3

	1.732
1	$\bar{3}$. $\overline{00}$ $\overline{00}$ $\overline{00}$
+ 1	–1
27	200
+ 7	–189
343	1100
+ 3	–1029
3462	7100
	–6924
	176

$\sqrt{3} = 1.732$ (aprox)

(b) 11

	3.3166
3	$\overline{11}$. $\overline{00}$ $\overline{00}$ $\overline{00}$
+ 3	–9
63	200
+ 3	–189
661	1100
+ 1	661
6626	43960
	39756
	4144

$\sqrt{11} = 3.3166$ (aprox)

(c) 125

	11.180
1	$\bar{1}\ \overline{25}.\ \overline{00}\ \overline{00}$
+ 1	–1
21	025
+ 1	–21
221	400
+ 1	–221
2228	17900
+ 8	–17824
22360	7600
	– 000
	7600

$\sqrt{125} = 11.80$ (aprox)

(d) 3460

	58.821
5	$\overline{34}\ \overline{60}\ .\ \overline{00}\ \overline{00}$
+ 5	–25
108	960
+ 8	–864
1168	9600
+ 8	–9344
11762	25600
	–23524
	2076

$\sqrt{3460} = 58.821$ (aprox)

4. Area of square = side × side

$\therefore$ $(\text{side})^2 = 425.5969\ \text{m}^2$

$\therefore$ side $= \sqrt{425.5969}\ \text{m}$

$= 20.63$

Length of each side = 20.63

	20.63
2	$\overline{425}\ .\ \overline{59}\ \overline{69}$
+ 2	–4
40	025
	0
406	2559
+ 6	–2436
4123	12369
	–12369
	0

5. Side of square = 8

$(AD)^2 = (AB)^2 + (BD)^2$ (Pythagorean triplet)

$(AD)^2 = (8)^2 + (8)^2$

$= 64 + 64$

$AD = \sqrt{128}$

$= 11.31$ cm.

	11.31
1	128.00 00
+ 1	–1
21	28
+ 1	–21
223	700
+ 3	–669
2261	3100
	–2261
	839