(b) $72x^2 y^3$ by $-2xy$

$$72x^2 y^3 \div -2xy = \frac{\overset{36}{72} \times x \times x \times y \times y \times y}{-2 \times x \times y} = -36xy^2.$$

(c) $-13x^3 yz^3$ by $-5xyz^2$

$$-13x^2 yz^3 \div (-5xyz^2) = \frac{-13x \times x \times y \times z \times z \times z}{-5 \times x \times y \times z \times z} = \frac{13x^2}{5}.$$

4. (a) $16x^4 + 12x^3 + 8x^2 + 4x$ by $4x$

$$(16x^4 + 12x^3 + 8x^2 + 4x) \div 4x = \frac{16x^4}{4x} + \frac{12x^3}{4x} + \frac{8x^2}{4x} + \frac{4x}{4x}$$

$$= 4x^3 + 3x^2 + 2x + 1$$

(b) $8x^4 yz - 5xy^3 z + 24x^2 y^4$ by $3xyz$

$$(8x^4 yz - 5xy^3 + 24x^2 y^4) \div 3xyz = \frac{8x^4 yz}{3xyz} - \frac{-5xy^3 z}{3xyz} + \frac{24x^2 y^4}{3xyz}$$

$$= \frac{8}{3}x^3 - \frac{5}{3}y^2 + 8xy^3 z.$$

(c) $9x^2 y - 6xy + 12xy^2$ by $\frac{-3}{2}xy$

$$(9x^2 y - 6xy + 12xy^2) \div \frac{-3}{2}xy = \frac{9x^2 y}{\frac{-3}{2}xy} - \frac{6xy}{\frac{-3}{2}xy} + \frac{12xy^2}{\frac{-3}{2}xy}$$

$$= (-6) - (-4) + (-8y)$$

$$= -6x - 8y + 4.$$

5. (a) $-14x^2 - 13x + 12 \div 2x + 3$

$$\begin{array}{r|l} & -7x + y \\ \hline 2x + 3 & -14x^2 - 13x + 12 \\ & -14x^2 - 21x \\ & + \qquad + \\ \hline & 8x + 12 \\ & -8x + 12 \\ \hline & 00 \end{array}$$

Quotient = $-7x + 4$, Remainder = 0

Check : Dividend = Divisor × quotient + Remainder.

$= (2 + 3) \times (-7x + 4) + 0$

$= -14x^2 + 8x - 21x + 12$

$= -14x^2 - 13x + 12$

= Dividend

(b) $6y^5 - 28y^3 + 3y^2 + 30y - 9$ by $2y^2 - 6$

$$\begin{array}{r|l} & 3y^3 - 5y + \frac{3}{2} \\ \hline 2y^2 - 6 & 6y^5 - 2y^3 + 3y^2 + 30y - 9 \\ & 6y^5 - 18y^3 \\ & - \quad + \\ \hline & -10y^3 + 3y^2 + 30y - 9 \\ & -10y^3 \qquad + 30y \\ & + \qquad - \\ \hline & 3y^2 - 9 \\ & 3y^2 - 9 \\ & - \quad + \\ \hline & 0 \end{array}$$

Quotient $3y^3 - 5y + \frac{3}{2}$, Remainder = 0

Dividend = Divisor × Quotient + Remainder

$= (2y^2 - 6) \times \left(3y^3 - 5y + \frac{3}{2}\right) + 0$

$= 6y^5 - 10y^3 + 3y^2 - 18y^3 - 30y - 9$

= Dividend.

(c) $9x^2y - 6xy + 12xy^2$ by $\frac{-3}{2}xy$

$$\begin{array}{r|l} & -6x + 4 - 8y \\ \hline \frac{-3}{2}xy & 9x^2y - 6xy + 12xy^2 \\ & +9x^2y \\ & - \\ \hline & -6xy + 12xy^2 \\ & -6xy \\ & + \\ \hline & 12xy^2 \\ & -12xy^2 \\ \hline & 00 \end{array}$$

Quotient $= -6x + 14 - 2y$, Remainder = 0

Check : Dividend = Divisor × Quotient + Remainder

$= \left(\frac{-3}{2}xy\right) \times (-6x + 4 - 8) + 0$

$= +9x^2y - 6xy + 12xy^2 + 0$

$= 9x^2y - 6xy + 12xy^2$

= Dividend.

(d) $-12x^4 - 22x^3 - 10x^2 + 34x - 75$ by $3x + 7$

$$
\begin{array}{r}
-4x^3 + 2x^2 - 8x + 30 \\
3x + 7\overline{\smash{)}\,-12x^4 - 22x^3 - 10x^2 + 34x - 75} \\
\underset{-}{-12x^4} \underset{+}{- 8x^3} \qquad\qquad\qquad\qquad \\
\hline
6x^3 - 10x^2 + 34x - 75 \\
\underset{-}{6x^3} \underset{-}{- 10x^2} \qquad\qquad\quad \\
\hline
-24x^2 + 34x - 75 \\
\underset{+}{-24x^2} \underset{+}{- 56x} \qquad \\
\hline
90x - 75 \\
\underset{-}{90x} \underset{-}{+ 210} \\
\hline
-285
\end{array}
$$

Quotient $= -4x^3 + 2x^2 - 8x + 30$, Remainder = 285

Check : Dividend = Divisor × Quotient + Remainder

$= (3x + 7)(-4x^3 + 2x^2 - 8x + 30) + (-285)$

$= (-12x^4 + 6x^3 - 24x^2 + 90x - 28x^3 + 14x^2 - 56x + 210) + (-285)$

$= -12x^4 - 22x^3 - 10x^2 + 34x + 210 - 285$

$= -12x^4 - 22x^3 - 10x^2 + 34x - 75 =$ Dividend.

6. (a) If $(3x - 1)$ is a factor of $6x^2 + x - 1$, then it is divide the $6x^2 + x - 1$ completely.

Check :

$$
\begin{array}{r}
2x + 1 \\
3x - 1\overline{\smash{)}\,6x^2 + x - 1} \\
\underset{-}{6x^2} \underset{+}{- 2x} \quad\;\; \\
\hline
3x - 1 \\
\underset{+}{-3x} \underset{+}{- 1} \\
\hline
0
\end{array}
$$

Hence, $3x - 1$ is a factor of $6x^2 + x - 1$.

(b) If $2x - 5$ is a factor of $4x^4 - 10x^3 - 10x^2 + 30x - 15$, then it is complely divide the $4x^4 - 10x^3 - 10x^2 + 30x - 15$.

Check :

$$
\begin{array}{r}
2x^3 - 5x \\
2x - 5\overline{\smash{)}\,4x^4 - 10x^3 - 10x^2 + 30x - 15} \\
\underset{-}{4x^4} \underset{+}{- 10x^3} \qquad\qquad\qquad\quad \\
\hline
-10x^2 + 30x \quad\;\; \\
\underset{+}{-10x^2} \underset{-}{+ 25x} \quad\;\; \\
\hline
5x - 15 \quad \text{Remainder}
\end{array}
$$

Hence $2x - 5$ is not a factor of $4x^4 - 10x^3 - 10x^2 + 30x - 15$.

Exercise 7.2

1. (a) $(5x+3y)^2$

$\because \quad (a+b)^2 = a^2 + b^2 + 2ab$

Here, $a = 5x$ and $b = 3y$

$\therefore \quad (5x+3y)^2 = (5x)^2 + (3y)^2 + 2(5x)(3y)$

$= 25x^2 + 9y^2 + 30xy.$

(b) $(5x+12x^2)^2$

Here $a = 5$ and $b = 12x^2$

$\because \quad (a+b)^2 = a^2 + b^2 + 2ab\,(5+12x^2)^2 + (5)^2 + (12x^2)^2 + 2(5)(12x^2)$

$= 15 + 124^4 + 120x^2.$

(c) $\left(5x+\frac{1}{5y}\right)^2$

Here $a = 5x$, $b = \frac{1}{5y}$

$\because \quad \left(5x+\frac{1}{5y}\right)^2 = (5x)^2 + \left(\frac{1}{5y}\right)^2 + 2(5x)\left(\frac{1}{5y}\right)$

$= 25x^2 + \frac{1}{25y^2} + 2x \times \frac{1}{y}.$

(d) $(3x-4y)^2$

Here, $a = 3x$, $b = 4y$

$\because \quad (a-b)^2 = a^2 + b^2 - 2ab$

$\therefore \quad (3x-4y)^2 = (3x)^2 + (4y)^2 - (2)(3x)(4y)$

$= 9x^2 + 16y^2 - 24xy.$

(e) $\left(\sqrt{3x} - \frac{1}{5}y\right)^2$

Here $a = \sqrt{3}x$, $b = \frac{1}{5}y$

$\because \quad (a-b)^2 = a^2 + b^2 - 2ab$

$\therefore \quad \left(\sqrt{3}x - \frac{1}{5}y\right)^2 = (\sqrt{3}x)^2 + \left(\frac{1}{5}y\right)^2 - 2(\sqrt{3}x)\left(\frac{1}{5}y\right)$

$= 3x^2 + \frac{1}{25}y^2 - \frac{2\sqrt{3}}{5}xy.$

(f) $(x-3y)\times(x-3y)$

$(x-3y)\times(x-3y) = (x-3y)^2$

$= (x)^2 + (3y)^2 - 2(x)(3y)$

$= x^2 + 9y^2 - 6xy.$

2. (a) $9x^2 + 49y^2 + 42xy$

Here $x = 3$ and $y = 1$

$$9x^2 + 49y^2 + 42xy = (3x)^2 + (7y)^2 + 2 \times 3x \times 7y$$
$$= (3x + 7y)^2 = 3 \times 3 + 7 \times 1)^2 \quad \text{(when } x = 3,\ y = 1)$$
$$= (9 + 7)^2 = (16)^2 = 256.$$

(b) $25^2 + 64y^2 - 80xy \quad x = 4,\ y = z$

$$25x^2 + 64y^2 - 80xy = (5x)^2 + (8y)^2 - (2)(5x)(8y)$$
$$= (5x - 8y)^2\ (5 \times 4 - 8 \times 2)^2 \qquad (\because x = 4 \text{ and } y = 2)$$
$$= (20 - 16)^2 = (4)^2 = 16.$$

3. (a) $2x + 3y = 8$ On squaring both sides

$$(2x + 3y)^2 = (8)^2$$
$$4x^2 + 9y^2 + 2 \times 2x \times 3y = 64$$
$$4x^2 + 9y^2 + 12(xy) = 64$$
$$4x^2 + 9y^2 + 12(2) = 64 \qquad (\because xy = 2)$$
$$4x^2 + 9y^2 = 64 - 24$$
$$4x^2 + 9y^2 = 40.$$

(b) $3x - 7y = 8$ on squaring both sides

$$(3x - 7y)^2 = (8)^2$$
$$9x^2 + 49y^2 - 2 \times 3x \times 7y = 64\ 9x^2 + 49y^2 - 42(xy) = 64$$
$$9x^2 + 49y^2 - 42(-1) = 64\ 9x^2 + 49y^2 + 42 = 64$$
$$9x^2 + 49y^2 = 64 - 42$$
$$9x^2 + 49y^2 = 22.$$

4. (i) $x + \frac{1}{x} = 6$ squaring on both side

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$
$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 36$$
$$x^2 + \frac{1}{x^2} + 2 = 36$$
$$x^2 + \frac{1}{x^2} = 36 - 2$$
$$x^2 + \frac{1}{x^2} = 34.$$

(ii) $x^2 + \frac{1}{x^2} = 34$ squaring on both side

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 1156$$

$$x^4 + \frac{1}{x^4} + 2 = 1156$$

$$x^4 + \frac{1}{x^4} = 1156 - 2 = 1154.$$

5. (a) $\frac{x-1}{x} = 5$ squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = (5)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 25$$

$$x^2 + \frac{1}{x^2} = 25 + 2$$

$$x^2 + \frac{1}{x^2} = 27.$$

(b) $x^2 + \frac{1}{x^2} = 27$ squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (27)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \frac{1}{x^2} = 729$$

$$x^4 + \frac{1}{x^4} + 2 = 729$$

$$x^4 + \frac{1}{x^4} = 729 - 2$$

$$x^4 + \frac{1}{x^4} = 727.$$

6. (a) $(4x + 4y)(4x - 5y) = 16x^2 - (5y \times 4x) + 16xy - 20y^2$

$= 16x^2 - 20xy + 16xy - 20y^2$

$= 16x^2 - 20y^2 - 4xy.$

(b) $(ab + cd)(ab - cd) = (ab \times ab) - (ab \times cd) + (cd \times ab) - (cd \times cd)$

$= a^2b^2 - abcd + abcd - c^2d^2$

$= a^2b^2 - c^2d^2 = (ab)^2 - (d)^2$

(c) $(x-1)(x+1)(x^2+1)(x^4+1) = (x^2 + x - x - 1)$

$= (x^6 + x^2 + x^4 + 1)$

$= (x^2 - 1)(x^6 + x^2 + x^4 + 1)$

$= x^8 + x^4 + x^6 + x^2 - x^6 - x^2 - x^4 - 1$

$= x^8 - 1.$

(d) $\left(x+\frac{y}{5}-1\right)\left(x+\frac{y}{5}+1\right)=x^2+\frac{xy}{5}+x+\frac{xy}{5}+\frac{y^2}{25}+\frac{y}{5}-x-\frac{y}{5}-1$

$=x^2+\frac{y^2}{25}+\frac{2xy}{5}-1.$

7. (a) $(103)^2$ $(103)^2=(100+3)^2$

We know that $(a+b)^2=a^2+b^2+2ab$

then $(100+3)^2=(100)^2+(3)^2+2(100)(3)$

$=10000+9+600$

$=106009.$

(b) $(91)^2$ $(91)^2=(100-9)^2$

We know that, $(a-b)^2=a^2+b^2-2ab$

$(100-9)^2=(100)^2+(9)^2-2(100)\times(9)$

$10000+81-1800=10081-1800=8281.$

(c) $(0.98)^2=(1-0.02)^2$

then $(1-0.02)^2=(1)^2+(0.02)^2-2(1)\times(0.02)$

$=1+0.0004-0.04=0.9604.$

(d) $(97)^2$ $(97)^2=(100-3)^2$

$\therefore$ $(100-3)^2=(100)^2+(3)^2-2\times100\times3$

$=10000+9-600$

$=10009-600=9409.$

(e) 103×97

We know that, $a^2-b^2=(a+b)(a-b)$

$103\times97=(100+3)(100-3)=(100)^2-(3)^2$

$=10000-9=9991.$

(f) $104\times104\,(100)^2=(100+4)^2$

$=(100)^2+(4)^2+2\times100\times4$

$=10000+16+800=10816.$

(g) $166\times166-134\times134=(166)^2-(134)^2$

$=(166+134)(166-134)$ $[\because a^2-b^2=(a+b)(a-b)]$

$=(300)(32)=9600.$

(h) $0.78\times0.78-0.22\times0.22$

$(0.78)^2-(0.22)^2=(0.78+0.22)(0.78-0.22)$ $[\because a^2-b^2=(a+b)(a-b)]$

$=(1)(0.56)=0.56.$

(i) $0.54\times0.54-0.46\times0.46=(0.54)^2-(0.46)^2$

$(0.54)^2-(0.46)^2=(0.54+0.6)(0.54-0.46)$

$=(1.00)(0.08)=0.08.$

Exercise 7.3

1. (a) $(x-2y-5z)^2$

$\because$ $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$

$$(x-2y-5z)^2 = (x)^2 + (-8y)^2 + (-5z)^2 + 2(x \times -2y)$$
$$+ 2(-2y) \times (-5z) + 2(x)(-5z)$$
$$= x^2 + 4y^2 + 25z^2 - 4xy + 20yz - 10xz$$
$$= x^2 + 4y^2 + 25z^2 - 4xy + 20yz - 10xz.$$

(b) $\left(\frac{1}{4}x - \frac{1}{2}y + 16\right)^2$

$\because \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\therefore \quad \left(\frac{1}{4}x - \frac{1}{2}y + 16\right)^2 = \left(\frac{1}{4}x\right)^2 + \left(-\frac{1}{2}y\right)^2 + (16)^2 + \left(2 \times \frac{1}{4}x \times \frac{-1}{2}y\right)$$
$$+ 2 \times \left(\frac{-1}{2}y\right) \times (16) + 2(16)\left(\frac{1}{4}x\right)$$
$$= \frac{1}{16}x^2 + \frac{1}{4}y^2 + 256 - \frac{1}{y}xy - 16y + 8x.$$

(c) $\left(\frac{a}{b} + \frac{b}{c} + \frac{c}{d}\right)^2$

$\because \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\text{then} \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{d}\right)^2 = \left(\frac{a}{b}\right)^2 + \left(\frac{b}{c}\right)^2 + \left(\frac{c}{d}\right)^2 + 2\left(\frac{a}{b} \times \frac{b}{c}\right)$$
$$+ 2\left(\frac{b}{c} \times \frac{c}{d}\right) + 2\left(\frac{c}{d} + \frac{a}{b}\right)$$
$$= \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} + 2\left(\frac{a}{c}\right) + 2\left(\frac{b}{d}\right) + 2\left(\frac{ac}{bd}\right).$$

2. $x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6xz$

$x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6xz$
$$= (x)^2 + (2y)^2 + (3z)^2 + 2(x \times 2y) + 2(2y \times 3z) + 2(3z \times x)$$
$$= (x + 2y + 3z)^2 \qquad (\because a + b + c^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$$

Now, on putting the value of x, y and z.

$x = 8,\ y = 7$ and $z = 6$

then $\quad (x + 2y + 3z)^2 = [8 + (2 \times 7) + (3 \times 6)]^2$
$$= (8 + 14 + 18)^2 = (40)^2 = 1600.$$

3. $x^2 + 4y^2 + 25z^2 - 4xy + 20yz - 10xz$
$$= (x)^2 + (-2y)^2 + (-5z)^2 + 2(x \times -2y) + 2(-2y \times 5z) + 2(-x \times 2)$$
$$= (x + (-2y) + (-5z)^2 = (x - 2y - 5z)^2$$
$$[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 12b + 2ca)]$$

Now, on putting the value of x, y and $x = 9$, $y = 2$ and $z = 1$.

then $(x - 2y - 5z)^2 = \{9 - (2 \times 2) - (5 \times 1)\}^2$
$$= [9 - 4 - 5]^2$$
$$= [9 - 9]^2 = [0]^2 = 0.$$

4. $x+y+z=12$ squaring both the sides.

$$(x+y+z)^2=(12)^2$$
$$x^2+y^2+z^2+2xy+2yz+2zx=144$$
$$(x^2+y^2+z^2)+(2xy+2yz+2zx)=144$$
$$\because \quad x^2+y^2+z^2=64$$
$$\therefore \quad 64+(2xy+2yz+22x)=14$$
$$2xy+2yz+2zx=144-64=80$$
$$2(xy+yz+zn)=80$$
$$xy+yz+zn=\frac{80}{2}=40.$$

5. $x+y+z=8$ squaring both sides.

$$(x+y+z)^2=(8)^2$$
$$x^2+y^2+z^2+2xy+2yz+2xz=64$$
$$x^2+y^2+z^2+2(xy+yz+xz)=64$$
$$x^2+y^2+z^2+2(13)=64 \qquad (\because xy+yz+xz=13)$$
$$x^2+y^2+z^2+26=64$$
$$x^2+y^2+z^2=64-26=38.$$

6. $x^2+y^2+z^2=35$ on using identify

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2xz$$
$$(x+y+z)^2=(x^2+y^2+z^2)+2(xy+yz+zx)$$
$$(x+y+z)=\sqrt{35+2(23)} \qquad (\because x^2+y^2+z^2=35 \text{ and } xy+yz+zx=23)$$
$$=\sqrt{35+46}=\sqrt{81} \qquad (\therefore\ x+y+z=9)$$

7. (a) $(2x+p-c)^2-(2x-p+c)^2$

$$\because \quad (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$
$$\therefore \quad (2x+p+(-c))^2-(2x+(-p)(c)^2$$
$$=(4x^2+p^2+c^2+4xp-2pc-4cx)-(4x^2+p^2+c^2-4xp-2pc+4xc)$$
$$=4x^2+p^2+c^2+4xp-2pc-4cx-4x^2-p^2-c^2+4xp+2pc-4xc$$
$$=8xp-8xc$$
$$=8x(p-c).$$

(b) $(x^2+y^2-z^2)^2-(x^2-y^2+z^2)^2$

$$\therefore \quad (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$
$$\therefore \quad (x^2+y^2+(z)^2-(x^2+(-y)^2+z^2)^2$$
$$=(x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2)$$
$$=-(x^4+y^4+z^4-2x^2y^2-2y^2z^2+2z^2x^2)$$
$$=x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2-x^4-y^4$$
$$=-z^4+2x^2y^2+2y^2z^2-2z^2x^2=4x^2y^2-4z^2x^2$$
$$=4x^2(y^2-z^2)$$
$$=4x^2(y+z)(y-z).$$

(c) $(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$

$\because \quad (a+b+c)^2 = a^2+b^2+2ab+2bc+2ca$

$\therefore \quad (a+b+c)^2+(a-b+c)^2+(a+b-c)^2$

$$= (a^2+b^2+2ab+2bc+2ca)+(a^2+b^2+c^2-2ab-2bc+2ca)$$
$$+(a^2+b^2+c^2+2ab-2bc-2ca)$$
$$= a^2+b^2+c^2+2ab+2bc+2ca+a^2+b^2+c^2-2ab-2bc+2ca$$
$$+a^2+b^2+c^2+2ab-2bc-2ca$$
$$= 3a^2+3b^2+3c^2+2ab+2ca-2bc$$
$$= 3(a^2-b^2+c^2)+2ab-2bc+2ca).$$

Exercise 7.4

1. (a) $(3x-2y)^3$

$\because \quad (a-b)^3 = a^3-b^3-3ab(a-b)$

$\therefore \quad (3x-2y)^3 = 27x^3-8y^3-3(3x\times 2y)(3x-2y)$

$$= 27x^3-8y^3-(18xy)(3x-2y) = 27x^3-8y^3-54x^2y+36xy^2.$$

(b) $\left(\frac{1}{3}x+\frac{5}{3}y\right)^3$

$\because \quad (a+b)^3 = a^3+b^3+3ab(a+b)$

$$\left(\frac{1}{3}x+\frac{5}{3}y\right)^3 = \frac{1}{27}x^3+\frac{125}{27}y^3+3\left(\frac{1}{3}x\times\frac{5}{3}y\right)\left(\frac{1}{3}x+\frac{5}{3}y\right)$$
$$= \frac{1}{27}x^3+\frac{125}{27}y^3+\frac{5}{3}xy\left(\frac{1}{3}x+\frac{5}{3}y\right)$$
$$= \frac{1}{27}x^3+\frac{125}{27}y^3+\frac{5}{9}x^2y+\frac{25}{9}xy^2.$$

(c) $\left(\frac{1}{3x}-\frac{2}{5y}\right)^3$

$\because \quad (a-b)^2 = a^3-b^3-3ab(a-b)$

$$\therefore \quad \left(\frac{1}{3x}-\frac{2}{5y}\right)^3 = \frac{1}{27x^3}-\frac{8}{125y^3}-3\left(\frac{1}{3x}\times\frac{2}{5y}\right)\left(\frac{1}{3x}-\frac{2}{5y}\right)$$
$$= \frac{1}{27x^3}-\frac{8}{125y^3}-\frac{2}{5xy}\left(\frac{1}{3x}-\frac{2}{5y}\right)$$
$$= \frac{1}{27x^3}-\frac{8}{125y^3}-\frac{2}{15x^2y}+\frac{4}{25xy^2}.$$

2. (a) $(1004)^3\, l\, (1004)^3 = (1000+4)^3$

$\because \quad (a+b)^3 = a^3+b^3+3ab(a+b)$

$\therefore \quad (1000+4)^3 = (1000)^3+(4)^3+3(1000\times 4)(1000+4)$

$$= 1000\times 1000\times 1000+64+12000(1004)$$
$$= 1000000064-12048000 = 1012048064.$$

(b) $(599)^3$ $(599)^3 = (600-1)^3$

$\because \quad (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$(600-1)^3 = (600)^3 - (1)^3 - 3\times(600\times 1)(600-1)$$
$$= 216000000 - 1 - 1800(599)$$
$$= 216000000 - 1 - 1078200$$
$$= 214921799.$$

(c) $(9.8)^3$ $(9.8)^3 = (10-0.2)^3$

$$= (10)^3 - (0.2)^3 - 3(10\times 0.2)(10-0.2)$$
$$= 1000 - 0.008 - 6(10-0.2)$$
$$= 1000 - 0.008 - 60 + 1.2$$
$$= 941.192.$$

(d) $(8.01)^3$ $(8.01)^3 = (8+0.01)^3$

$\because \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\therefore \ (8+0.01)^3 = (8)^3 + (0.01)^3 + 3(8\times 0.01)(8+0.01)$
$$= 512 + 0.000001 + 0.24\,(8.01)$$
$$= 512 + 0.000001 + 1.9224$$
$$= 513.922401.$$

3. $x + y = 5$

On cubing both sides.

$$(x+y)^3 = (5)^3$$
$$x^3 + y^3 + 3xy(x+y) = 125$$
$$x^3 + y^3 + 3(6)(5) = 125 \qquad (\because x + y = 5 \text{ and } xy = 6)$$
$$x^3 + y^3 + 90 = 125$$
$$x^3 + y^3 = 125 - 90 = 35.$$

4. $x + y = 12$

On cubing both sides

$$(x+y)^3 = (12)^3$$
$$x^3 + y^3 + 3xy(x+y) = 1728$$
$$x^3 + y^3 + 3\times 27(12) = 1728 \qquad (\because xy = 27 \text{ and } x + y = 12)$$
$$x^3 + y^3 = 972 = 1728$$
$$x^3 + y^3 = 1728 - 972 = x^3 + y^3 = 756.$$

5. $x - y = 4$

On cubing both sides.

$$(x-y)^3 = (4)^3$$
$$x^3 - y^3 - 3(xy)(x-y) = 64$$
$$x^3 - y^3 - 3(21)(4) = 64 \qquad (\because xy = 21, x - y = 4)$$
$$x^3 - y^3 - 252 = 64$$
$$x^3 - y^3 = 64 + 252$$
$$x^3 - y^3 = 316.$$

6. $3x - 2y = 11$

On cubing with sides.

$$(3x - 2y)^3 = (11)^3$$

$$(3x)^3 - (2y)^3 - 3(3x \times 2y)(3x - 2y) = 1331$$

$$27x^3 - 8y^3 - 18xy(3x - 2y) = 1331$$

$$27x^3 - 8y^3 - 18 \times 12 \times 11 = 1331 \qquad (\because xy = 12 \text{ and } 3x - 2y = 11)$$

$$27x^3 - 8y^3 = 1331 + 2376$$

$$27x^3 - 8y^3 = 3707.$$

7. $x + \frac{1}{x} = 7$

On cubing both sides.

$$\left(x + \frac{1}{x}\right)^3 = (7)^3$$

$$x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 343$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 343$$

$$x^3 + \frac{1}{x^3} + (3 \times 7) = 343 \qquad \left[\because x + \frac{1}{x} = 7\right]$$

$$x^3 + \frac{1}{x^3} = 343 - 21$$

$$x^3 + \frac{1}{x^3} = 322.$$

8. $x - \frac{1}{x} = 5$

On cubing both sides.

$$\left(x - \frac{1}{x}\right)^3 = (5)^3$$

$$\because \quad (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$x^3 - \frac{1}{x^3} - 3\left(x \times \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 125$$

$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 125$$

$$x^3 - \frac{1}{x^3} - 3 \times 5 = 125$$

$$x^3 - \frac{1}{x^3} = 125 + 15$$

$$x^3 - \frac{1}{x^3} = 140.$$

9. $x^2 + \frac{1}{x^2} = 7$

On putting

$$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \qquad \because\ x^2 + \frac{1}{x^2} = 7$$

$$\therefore\ \left(x+\frac{1}{x}\right)^2 = 7+2$$

$$\left(x+\frac{1}{x}\right)^2 = 9 x+\frac{1}{x} = 3$$

Now, on cubing the both sides

$$\left(x+\frac{1}{x}\right)^3 = (3)^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x \times \frac{1}{x}\right)\left(x-\frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + 3(3) = 27 \qquad \left(\because x - \frac{1}{x} = 3\right)$$

$$x^3 + \frac{1}{x^3} + 9 = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9$$

$$x^3 + \frac{1}{x^3} = 18.$$

10. $x^2 + \frac{1}{x^2} = 27$

On putting $\left(x-\frac{1}{-x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$$\because\ x^2 + \frac{1}{x^2} = 27$$

$$\therefore\ \left(x-\frac{1}{x}\right)^2 = 27-2 \quad \Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = 25$$

$$\left(x-\frac{1}{x}\right) = 5.$$

Now, on cubing both sides

$$\left(x-\frac{1}{x}\right)^3 = (5)^3$$

$$x^3 - \frac{1}{x^3} - 3\left(x \times \frac{1}{x}\right)\left(x-\frac{1}{x}\right) = 125$$

$$x^3 - \frac{1}{x^3} - 3 \times (5) = 125 \qquad \left[\because x - \frac{1}{x} = 5\right]$$

$$x^3 - \frac{1}{x^3} = 125 + 15$$
$$x^3 - \frac{1}{x^3} = 140.$$

11. (a) $(a-3b)^3 + (a+3b)^3$

$$= [a^3 - (3b)^3 - 3(a \times 3b)(a-3b)] + [a^3 + (3b)^3 + 3(a \times 3b)(a+3b)]$$
$$= [a^3 - 27b^3 - 9ab(a-3b)] + [a^3 + 27b^3 + 9ab(a+3b)]$$
$$= a^3 - \cancel{27b^3} - 9a^2b + \cancel{27ab^2} + a^3 + \cancel{27b^3} + 9a^2b + \cancel{27ab^2}$$
$$= 2a^3 + 54ab^2.$$

(b) $\left(\frac{1}{3}a + \frac{2}{3}b^3\right) - \left(\frac{1}{3}a - \frac{2}{3}b\right)^3$

$$= \left[\left(\frac{1}{3}a\right)^3 + \left(\frac{2}{3}b\right)^3 + 3\left(\frac{a}{3} \times \frac{2b}{3}\right)\left(\frac{a}{3} - \frac{2b}{3}\right)\right]$$
$$- \left[\left(\frac{1}{3}a^3\right) - \left(\frac{2}{3}b\right)^3 - 3\left(\frac{a}{3} \times \frac{2}{3}b\right)\left(\frac{a}{3} - \frac{2b}{3}\right)\right]$$
$$= \left[\frac{1 \times a^3}{27} + \frac{8b^3}{27} + \frac{2ab}{3}\left(\frac{a}{3} + \frac{2b}{3}\right)\right] - \left[\frac{a^3}{27} - \frac{8b^3}{27} - \frac{2}{3}ab\left(\frac{a}{3} - \frac{2b}{3}\right)\right]$$
$$= \frac{a^3}{27} + \frac{8b^3}{27} + \frac{2a^2b}{9} + \frac{4ab^2}{9} - \left[\frac{a^3}{27} - \frac{8b^3}{27} - \frac{2a^2b}{9} + \frac{4ab^2}{9}\right]$$
$$= \cancel{\frac{a^3}{27}} + \frac{8b^3}{27} + \frac{2a^2b}{9} + \cancel{\frac{4ab^2}{9}} - \cancel{\frac{a^3}{27}} + \frac{8b^3}{27} + \frac{2a^2b}{9} - \cancel{\frac{4ab^2}{9}}$$
$$= 2\left(\frac{8b^3}{27}\right) + 2\left(\frac{2a^2b}{9}\right) = \frac{16b^3}{27} + \frac{4a^2b}{9}.$$

Exercise 7.5

1. (a) $(x+2)(x+3)$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$(x+2)(x+3) = x^2 + (3x+2)x + 6 = x^2 + 5x + 6.$

(b) $(x-3)(x+7)$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$(x-3)(x+7) = x^2 + (7+(-3)x - 21$

$= x^2 + 4x - 21.$

(c) $(3x+4)(3x-6)$

$(3x+4)(3x-6) = 9x^2 - 18x + 12x - 24$

$= 9x^2 - 6x - 24.$

(d) $(2x-1)(2x-7)$

$(2x-1)(2x-7) = 4x^2 - 14x - 2x + 7$

$= 4x^2 - 16x + 7.$

2. (a) 56×48

$$56 \times 48 = (50+6)(50-2)$$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore \quad (50+6)(50-2) = (50)^2 + (6+(-2)50 + 6\times(-2)$

$= 2500 + (4)50 - 12$

$= 2500 + 200 - 12$

$= 2700 - 12 = 2688.$

(b) 95×97

$$95 \times 97 = (90+5)(90+7)$$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore \quad (90+5)(90+7) = (90)^2 + (5+7)90 + 35$

$= 8100 + 1080 + 35$

$= 9215.$

(c) 107×103

$$107 \times 103 = (100+7)(100+3)$$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore \quad (100+7)(100+3) = (100)^2 + (7+13)100 + 3\times 7$

$= 10000 + 1000 + 21 = 11021.$

(d) $35 \times 37\, 35 \times 37 = (30+5)(30+7)$

$\because \quad (x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore \quad (30+5)(30+7) = (30)^2 + (5+7)30 + 35$

$= 900 + 360 + 35 = 1295.$

3. (a) $(3x+4y)(9x^2 - 12xy + 16y^2)$

$= 27x^3 - \cancel{36x^2y} + \cancel{48xy^2} + \cancel{36x^2y} - \cancel{48xy^2} + 64y^3$

$= 27x^3 + 64y^3.$

(b) $(2x-3y)(4x^2 + 6xy + 9y^2)$

$= 8x^3 + \cancel{12x^2y} + \cancel{18xy^2} - \cancel{12x^2y} - \cancel{18xy^2} - 27y^3$

$= 8x^3 - 27y^3.$

(c) $(0.9x^2 + 0.7y)(0.81x^2 - 0.63xy + 0.49y^2)$

$= 0.729x^3 - 0.\cancel{567x^2}y + 0.\cancel{441}xy^2 + 0.\cancel{567}x^2y - 0.\cancel{441}xy^2 + 0.343y^3$

$= 0.729x^3 + 0.343y^3.$

4. (a) $(x+y+2z)(x^2 + y^2 + 4z^2 - xy - 2yz - 2zx)$

$= x^3 + xy^2 + 4xz^2 - x^2y - 2xyz - 2zx^2 + yx^2 + y^3 + 4yz^2 - xy^2 - 2y^2z$

$-2xyz + 2x^2z + 2y^2z + 8z^3 - 2xyz - 4yz^2 - 4z^2x$

$= x^3 + y^3 + 8z^3 - 6xyz.$

(b) $(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

$= 8x^3 + \cancel{2xy^2} + \cancel{18xz^2} + \cancel{4x^2y} + 6xyz - \cancel{12x^2}z - \cancel{4x^2}y - y^3 - \cancel{9yz^2} - \cancel{2xy^2}$

$- \cancel{3y^2}z + 6xyz + \cancel{12x^2}z + \cancel{3y^2}z + 27z^3 + 6xyz + \cancel{9yz^2} - \cancel{18xz^2}$

$= 8x^3 - y^3 + 27z^3 + 18xyz.$

(c) $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

$$= 8a^3 + \cancel{18ab^2} + \cancel{8ac^2} + \cancel{12a^2b} - 12abc + \cancel{8a^2c} - \cancel{12a^2b} - 27b^3 - \cancel{12bc^2}$$
$$- \cancel{18ab^2} + \cancel{18b^2c} - 12abc - \cancel{8a^2c} - \cancel{18b^2c} - 8c^3$$
$$- 12abc + \cancel{12bc^2} - \cancel{8c^2a}$$
$$= 8a^3 - 27b^3 - 8c^3 - 8c^2a.$$

5. (a) $(2x-3y)^3 + 3y-4z)^3 + (4z-2x)^3 = 3(2x-3y)(3y-4z)(4z-2x)$

L.H.S. $(2x)^3 - (3y)^3 - 3\times 6xy(2x-3y) + (3y)^2 - (4z)^3 - 3\times 3y$
$$\times 4z(3y-4z) + (4z)^3 - (2x)^3 - 3\times 8xz(4z-2x)$$
$$= \cancel{8x^3} - \cancel{27y^3} - 18xy(2x-3y) + \cancel{27y^3} - \cancel{64z^3} - 36yz(3y-4z)$$
$$+ \cancel{64z^3} - \cancel{8x^3} - 24xz(4z-2x)$$
$$= -18xy(2x-3y) - 36yz(3y-4z) - 24xz(4z-zx)$$
$$= 36x^2y + 54xy^2 - 108y^2z + 144yz^2 - 96xz^2 + 48x^2z$$

R.H.S. $3(2x-3y)(3y-4z)(4z-2x)$
$$= (6x-9y)(3y-4z)(4z-2x)$$
$$= (18xy - 24xz - 27y^2 + 36yz)(4z-2x)$$
$$= \cancel{72xyz} - 96xz^2 - 108y^2z + 14yz^2 - 36x^2y$$
$$+ 48x^2z + 54y^2x - \cancel{72xyz}$$
$$= -36x^2y + 54y^2x - 108y^2z + 144yz^2 - 96xz^2 + 48x^2z$$

Hence **L.H.S. = R.H.S.**

(b) $(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3 = 3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

L.H.S. $\cancel{a^6} - \cancel{b^6} - 3a^2b^2(a^2-b^2) + \cancel{b^6} - \cancel{c^6} - 3(b^2c^2)(b^2-c^2)$
$$+ \cancel{c^6} - \cancel{a^6} - 3c^2a^2(c^2-a^2)$$
$$= -3a^4b^2 + 3a^2b^4 - 3b^4c^2 + 3b^2c^4 - 3c^4a^2 + 3c^2a^4$$

R.H.S. $3(a^2-b^2)(b^2-c^2)(c^2-a^2)$
$$= (3a^2-3b^2)(b^2-c^2)(c^2-a^2)$$
$$= (3a^2b^2 - 3a^2c^2 - 3b^4 + 3b^2c^2)(c^2-a^2)$$
$$= \cancel{3a^2b^2c^2} - 3a^2c^4 - 3b^4c^2 + 3b^2c^4 - 3a^4b^2$$
$$+ 3a^4c^2 + 3a^2b^4 - \cancel{3b^2a^2c^2}$$
$$= -3a^4b^2 + 3a^2b^4 - 3b^4c^2 + 3b^2c^4 - 3c^4a^2 + 3c^2a^4$$

L.H.S. = R.H.S. **Proved**

6. (a) $(28)^3 - (78)^3 + (50)^3$

Let $a = 28$, $b = -78$ and $c = 50$

Then $a+b+c = 28 - 78 + 50 = 78 - 78 = 0$

$\because$ $a+b+c = 0$

$\therefore$ $a^3+b^3+c^3 = 3ab.$

$\therefore$ $(28)^3 + (-78)^3 + (50)^3 = -3\times 109200 = -327600$

then $(28)^3 - (78)^3 + (50)^3 = -327600.$

(b) $(7.8)^3 - (10.3)^3 + (2.5)^3$

Let, $a = 7.8$, $b = 10.3$ and $c = 2.5$

Then, $a + b + c = 7.8 + (-10.3) + 2.5$

$10.3 - 10.3 = 0$

$\because \quad a + b + c = 0$

$\therefore \quad a^3 + b^3 + c^3 = 3ab$

Now $(7.8)^3 + (-10.3)^3 + (2.5)^3 = 3 \times 7.8 \times -10.3 \times 2.5$

$= -602.55$

Hence $(7.8)^3 - (10.3)^3 + (2.5)^3 = -602.55$

MCQ's

1.(b) **2.** (a) **3.** (c) **4.** (d) **5.** (c) **6.** (a) **7.** (b) **8.** (d) **9.** (d) **10.** (b) **11.** (d) **12.** (d) **13.** (d).

Formative Assessment-2

1.(b) **2.** (c) **3.** (c) **4.** (c) **5.** (b) **6.** (b) **7.** (b) **8.** (c) **9.** (b) **10.** (c) **11.** (c) **12.** (b) **13.** (b) **14.** (b) **15.** (d) **16.** (c) **17.** (c) **18.** (d) **19.** (b) **20.** (c).

Summative Assessment-1

Section-A

Ans. 1. (d) **2.** (a) **3.** (b) **4.** (b) **5.** (b) **6.** (b) **7.** (c) **8.** (d).

Section-B

9. $4(1 - P) = 3(P - 2)$

$4 - 4P = 3P - 6$

$-4P - 3P = -6 - 4$

$-7P = -10$

$P = \frac{-10}{-7} = \frac{10}{7}$

10. $x - \frac{1}{x} = 4$ On squaring both sides

$\left(x - \frac{1}{x}\right)^2 = (4)^2$

$x^2 + \frac{1}{x^2} - 2 \times \frac{x}{x} = 16$

$x^2 + \frac{1}{x^2} = 16 + 2$

$x^2 + \frac{1}{x^2} = 18$

11. $\sqrt[3]{-2197}$

then $\sqrt[3]{-2197} = \sqrt[3]{-13 \times -13 \times -13}$

$= -13$

13	2197
13	169
13	13
	1

12. $2\frac{8}{9}$

Then $\sqrt{\frac{26}{9}} = \frac{\sqrt{26}}{\sqrt{3\times 3}}$

$= \frac{\sqrt{26}}{3}$

$= \frac{5.099}{3} = 1.699$

$= 1.70$ (approx)

	5.099
5	260000
+ 5	– 25
100	100
+ 0	000
10009	100000
+ 9	9081
1018	91900
	91701
	199

13. $\frac{1}{243}$

Prime factors of $243 = 3\times 3\times 3\times 3\times 3$

$= 3^5$

Then $\frac{1}{243} = \frac{1}{35}$

$= 3^{-5}$

3	243
3	81
3	27
3	9
3	3
	1

14. 1257

We know that, if the sum of the digits of the number is divisible by 3 the number must be divisible by 3.

So, sum of the digits $= 1+2+5+7= 15$

$\therefore$ 15 is divsible by 3

Hence, 1257 is also divisible by 3.

Section-C

15. First add

$$\frac{-3}{10}+\frac{5}{8}=\frac{-12+25}{40}=\frac{+13}{40}$$

then, $$\frac{4}{15}+\frac{2}{-5}=\frac{4-6}{15}=\frac{-2}{15}$$

Now subtract, $$\frac{-2}{15}-\frac{13}{40}=\frac{-16-39}{120}=\frac{-55}{120}=\frac{-11}{24}$$

16. Let q_1 and q_2 are two rational numbers between $\frac{-2}{3}$ and $\frac{1}{2}$

then $$q_1=\frac{1}{2}\left(\frac{-2}{3}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{-4+3}{6}\right)=\frac{1}{2}\left(\frac{-1}{6}\right)=\frac{-1}{12}$$

Now $$q_2=\frac{1}{2}\left(q_1+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{-1}{12}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{-1+6}{12}\right)=\frac{1}{2}\left(\frac{5}{12}\right)=\frac{5}{24}.$$

17.

$$\begin{array}{r} 3\ \ 0\ \ a \\ +\,b\ \ 6\ \ 3 \\ \hline 8\ \ c\ \ 1 \\ \hline \end{array}$$

Step 1. $\because\ 1 < 3$

$\therefore\ a + 3 = 11$

$a = 11 - 3 = 8$

Step 2. $1 + 0 + 6 = C$

$C = 7$

Step 3. $3 + b = 8$

$b = 8 - 3 = 5$

Hence, $a = 8, b = 5$ and $c = 7$

On putting the values of a, b and c

$$\begin{array}{r} 3\ \ 0\ \ \mathbf{8} \\ +\,\mathbf{5}\ \ 6\ \ 3 \\ \hline 8\ \ \mathbf{7}\ \ 1 \\ \hline \end{array}$$

18. $\left[\left(\frac{1}{4}\right)^{-2} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{2}\right)^{-3}$

$$\left\{\left(\frac{1}{4}\right)^{-2} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{2}\right)^{-3} = \left\{\left(\frac{4}{1}\right)^{2} - \left(\frac{3}{1}\right)^{3}\right\} \div \left(\frac{2}{1}\right)^{3}$$

$$= \{16 - 27\} \div 8 = \{-11\} \times \frac{1}{8} = \frac{-11}{8}$$

19. Let the number be x

Now according to question

$$\left(\frac{-3}{2}\right)^{-3} \div x = \left(\frac{9}{4}\right)^{-2}$$

$$\left(\frac{-8}{27}\right)^{3} \times \frac{1}{x} = \left(\frac{4}{9}\right)^{2} \quad \frac{-8}{27} \times \frac{1}{x} = \frac{16}{81}$$

$$\frac{-8}{27} \times \frac{1}{x} = \frac{16}{81} \quad \frac{-8}{27x} = \frac{16}{81}$$

or $$16 \times 27x = 81 \times -8$$

$$x = \frac{381 \times -8}{16 \times 27}$$

2

$$x = \frac{-3}{2}$$

20. Greatest 6-digit number = 999999

We need to find the least number that must be subtracted from 999999 to make it a perfec square.

Thus, the least number to be subtracted = 1998

Hence, the greatest 6-digit number which is a perfect square is $999999 - 1998 = 998001$

	999
9	$\overline{99}\,\overline{99}\,\overline{99}$
+9	– 81
189	1899
+9	–1701
1989	19899
	– 17901
	1998

21. $\sqrt[3]{\dfrac{-2197}{9261}}$

Writing 2197 and 9261 as a product of their prime factors, we get

$2197 = \overline{13 \times 13 \times 13}$

$9261 = \overline{7 \times 7 \times 7} \times \overline{3 \times 3 \times 3}$

$\therefore \quad \sqrt[3]{\dfrac{-2197}{9261}} = \sqrt[3]{\dfrac{\overline{-13 \times -13 \times -13}}{\overline{7 \times 7 \times 7} \times \overline{3 \times 3 \times 3}}}$

$= \dfrac{-13}{7 \times 3} = \dfrac{-13}{21}.$

13	2197
13	169
13	13
	1

7	9261
7	1323
7	189
3	27
3	9
3	3
	1

22. $3x^2 - 4x + 6x^3 - 5 + 8x^3 - 4x^2 + 5x + 5$

$= 3x^2 - 4x^2 - 4x + 5x + 6x^3 + 8x^3$

$= -x^2 + x + 14x^3 = 14x^3 - x^2 + x.$

23.
$$\begin{array}{r} 3x^2 + 1 \\ 1 + 4x \overline{)\,3x^2 + 12x^3 + 4x + 1} \\ \underset{-}{3x^2} + \underset{-}{12x^3} \qquad\qquad \\ \hline 4x + 1 \\ \underset{-}{4x} + \underset{-}{1} \\ \hline 00 \end{array}$$

$\therefore$ Quotient $= 3x^2 + 1$, Remainder $= 0$.

24. $\dfrac{2x}{3} - \dfrac{x}{5} = \dfrac{3x - 11}{4}$

$\dfrac{10x - 3x}{15} = \dfrac{3x - 11}{4}$ (cross multiplication)

$40x - 12x = 45x - 165$

$28x = 45x - 165$

$45x - 28x = 165$

$17x = 165$

$x = \dfrac{165}{17}.$

Section-D

25. Suppose Mohan's age $= x$ years

$\therefore$ Mohan's mother age $= 2x$ years

Mohan's age before 10 years $= x - 10$ years

Mohan's mother age before 10 years $= 2x - 10$ years

According to question

$(2x - 10) = 20 + (x - 10)$

then $2x - 10 = 20 + x - 10$

$2x - x = 20 - 10 + 10$

$x = 30 - 10 = 20$ years.

$\therefore$ Mohan's age = 20 years

and Mother's age = $2x = 2 \times 20 = 40$ years.

Hence Mohan's present age = 20 years

and Present age of Mohan's mother = 40 years.

26. $\because$ $(x+y)^2 = x^2 + y^2 + 2xy$

or $(x+y) = \sqrt{x^2 + y^2 + 2xy}$

$(x+y) = \sqrt{x^2 + y^2 + 2xy - 2xy + 2xy}$ (odd and subtract $2xy$)

$= \sqrt{x^2 + y^2 - 2xy + 4xy}$

$= \sqrt{(x-y)^2 + 4 \times (xy)}$

$= \sqrt{(12)^2 + 4 \times \frac{25}{4}} = \sqrt{144 + 25}$

$= \sqrt{169} = 13$

$\therefore$ $x + y = 13.$

27. $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$\because$ $a^2 - b^2 = a + b\,(a - b)$

then $(x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)(x+y)(x-y).$

28. $\sqrt[3]{0.00343} = \sqrt[3]{\frac{0.000343}{1.000000}}$

then writing 343 and 1000000

as a product of their prime factors, we get

7	343
7	49
7	7
	1

2	1000000
2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$343 = \overline{7 \times 7 \times 7}$

$1000000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

then $\sqrt[3]{\frac{343}{1000000}}$

$= \sqrt[3]{\frac{\overline{7 \times 7 \times 7}}{\overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{5 \times 5 \times 5} \times \overline{5 \times 5 \times 5}}}$

$= \frac{7}{2 \times 2 \times 5 \times 5}$

$= \frac{7}{100}$

$= 0.07.$

29. $\dfrac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}$

First we find square root

	7.7
7	$\overline{59}.\overline{29}$
+7	−49
147	1029
	−1029
	0

	2.3
2	$\overline{5}.\overline{29}$
+2	−4
43	129
	−129
	0

Now, $\dfrac{\sqrt{59.29}-\sqrt{5.29}}{\sqrt{59.29}+\sqrt{5.29}}$

$= \dfrac{7.7-2.3}{7.7+2.3}$

$= \dfrac{5.4}{10}$

$= 0.54$

30. $\left[\left(\dfrac{1}{3}\right)^{-3}-\left(\dfrac{1}{2}\right)^{-3}\right]\div\left(\dfrac{1}{4}\right)^{-3} = \left[\left(\dfrac{3}{1}\right)^{3}-\left(\dfrac{2}{1}\right)^{3}\right]\div\left(\dfrac{4}{1}\right)^{3}$

$= [(3)^3-(2)^3]\div[4]^3$

$= [27-8]\times\dfrac{1}{64}$

$= 19\times\dfrac{1}{64}=\dfrac{19}{64}.$

31. 63909

We know that, if the sum of the digits of the number is divisible by 9 then the number must be divisible by 9.

Now sum of the digits of the number $63909 = 6+3+9+0+9 = 27$

∵ 27 is divisible by 9.

Hence, the number 63909 is also divisible by 9.

32. Suppose the number $= x$

then according to question

$\left(\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{5}\right)+x=3$

$x = 3-\left(\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{5}\right)$

$x = \dfrac{3}{1}-\dfrac{1}{2}-\dfrac{1}{3}+\dfrac{1}{5}$

$x = \dfrac{90-15-10+6}{30}=\dfrac{96-25}{30}=\dfrac{71}{30}$

$x = \dfrac{71}{30}.$

33. $a = \dfrac{2}{-5}$ and $b = \dfrac{3}{4}$ then $|a+b| < |a|+|b|$

$|a+b| = \left|\dfrac{2}{-5}+\dfrac{3}{4}\right| = \left|\dfrac{-8+15}{20}\right|$

$= \left|\dfrac{7}{20}\right| = \dfrac{7}{20}$

Now $|a|+|b| = \left|\frac{2}{-5}\right| + \left|\frac{3}{4}\right|$

$= \frac{2}{5} + \frac{3}{4} = \frac{8+15}{20} = \frac{23}{20}$ Clearly, $\frac{7}{20} < \frac{23}{20}$

Hence, $|a+b| < |a|+|b|$ **Proved.**

34. $\because$ Price of 100 pens = ₹ $500\frac{3}{5}$ = ₹ $\frac{2503}{5}$

$\therefore$ Price of 1 pen = ₹ $\frac{2503}{5} \times \frac{1}{100}$

Price of 3 pens = ₹ $\frac{2503}{5} \times \frac{1}{100} \times 3$ = ₹ $\frac{2503 \times 3}{5 \times 100} = \frac{7509}{500}$

$\because$ Price of 50 pencils = ₹ $100\frac{1}{5}$ = ₹ $\frac{501}{5}$

$\therefore$ Price of 1 pencils = $\frac{\overset{1}{50}}{5} \times \frac{1}{50}$

Price of 5 pencils = ₹ $\frac{501 \times 5}{5 \times 50}$ = ₹ $\frac{2505}{250}$

Now the price of 3 pens and 5 pencils = ₹ $\left(\frac{7509}{500} + \frac{2505}{250}\right)$ = ₹ $\left(\frac{7509 + 5010}{500}\right)$

= ₹ $\left(\frac{12519}{500}\right)$ = ₹ 25.038

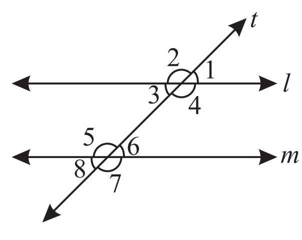
8. Lines and Angles

Exercise 8.1

1. Total no. of pairs of parallel lines = 6

$AB \parallel DE, AB \parallel FG, AB \parallel HI, DE \parallel EG, DE \parallel HI, FG \parallel HI.$

2. $m \parallel n$ Given



$\because$ $\angle 1 = 60°$

$\angle 1 + \angle 2 = 180$ (Linear pair)

$60 + \angle 2 = 180$ ($\because \angle 1 = 60°$)

$\angle 2 = 180 - 60$

$\angle 2 = 120°$

Now, $\angle 2 = \angle 5$ ($\because$ corresponding angles)

So, $\angle 5 = 120°$

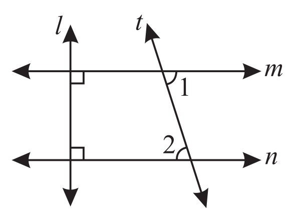
Then $\angle 5 + \angle 8 = 180°$ (Line pair)

$\therefore$ $120 + \angle 8 = 180$ ($\because \angle 5 = 120°$)

$\angle 8 = 180 - 120$

$\angle 8 = 60°$

3. $\because$ Line $m \perp l$ and Line $n \perp l$



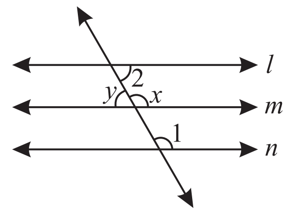
$\therefore$ Line $m \parallel$ line n

$\angle 1 = 180°$

$\therefore$ $\angle 1 = \angle 2$ ($\because$ Alternate angles)

$\therefore$ $\angle 2 = 80°$

4. $l \parallel m, m \parallel n,$



then $\angle 1 = \angle x$ ($\because$ Corresponding angle)

$\angle y + \angle x = 180$ (Linear pair)

$\angle y + 130 = 180$ ($\because \angle x = \angle 1 = 130$)

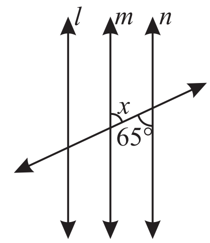
$\angle y = 180 - 130$

$\angle y = 50°$

Now $\angle y = \angle 2$ ($\because$ Alternate angles)

$\angle 2 = \angle y = 50°$

Hence $\angle 2 = 50°$.



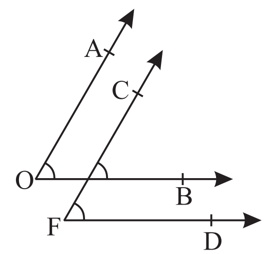
5. (a) $\because$ $l \parallel m$ and $m \parallel n$ (Given)

$\therefore$ $l \parallel n$

(b) $\because$ $m \parallel n$ (Given)

$\therefore$ $\angle x = \angle 65°$ (Alternate angle)

6. $\because$ $AO \parallel CF$



$\therefore$ $\angle AOB = \angle CMB$ ($\because$ Corresponding angle)

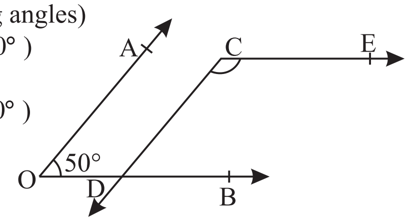
Now, $\because$ $OB \parallel FD$

$\therefore$ $\angle CMB = \angle CFD$ ($\because$ Corresponding angle)

$\because$ $\angle AOB = \angle CMB = \angle CFD$

$\therefore$ $\angle AOB = \angle CFD$ **Proved.**

7. $\because$ $AO \parallel DC$



$\therefore$ $\angle AOB = \angle CDB$ (Coresponding angles)

$\therefore$ $\angle CDB = 50°$ ($\because \angle AOB = 50°$)

Now $\angle CDB + \angle PDB = 180°$ (Linear pair)

$\angle 50 + \angle PDB = 180°$ ($\because \angle CDB = 50°$)

$\angle PDB = 180 - 50$

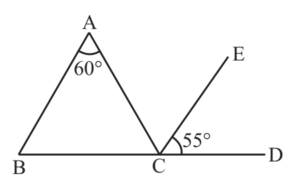
$\angle PDB = 130°$

Now, $\because$ $CE \parallel OB$

$\therefore$ $\angle PDB = \angle DCE$ (Corresponding angle)

$\therefore$ $\angle DCE = 130°$ ($\because \angle PDB = 130°$).

8. $\because$ $BA \parallel CE$



$\therefore$ $\angle BAC = \angle ACE$ (Alternate angle)

$\therefore$ $\angle ACE = 60$ ($\because BAC = 60°$)

Now $\angle BCA + \angle ACE + \angle ECD = 180°$ (Linear pair)

$\angle BCA + 60° + 55° = 180°$

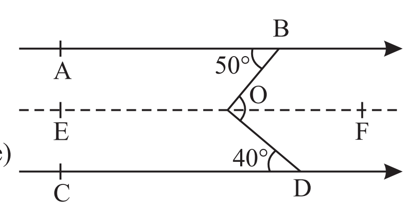
($\because \angle ACE = 60°$ and $\angle ECD = 55°$)

$\angle BCA = 180° - 115°$

$\angle BCA = 65°$

Hence $\angle BCA = 65°$.

9. $\because$ $AB \parallel CD$



Through D draw a line EOF parallel to line AB.

then $AB \parallel EF$

$\therefore$ $\angle ABO = \angle FOB$ ($\because$ Alternate angle)

$\therefore$ $\angle FOB = 50°$ ($\because \angle ABO = 50°$)

Now $\because$ $AB \parallel CD$

$\therefore$ $CD \parallel EF$

then $\angle CDO = \angle FOD$ (alternate angle)

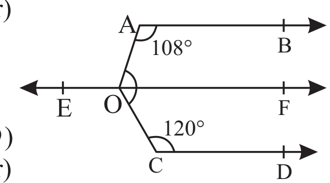
$\therefore$ $FOD = 40°$ ($\because \angle CDO = 40°$)

Now, $\angle BOD = \angle BOF + \angle FOD$

$\therefore$ $\angle BOD = 50 + 40$ ($\because \angle BOF = 50°$ and $\angle FOD = 40°$)

$\therefore$ $\angle BOD = 90°$.

10. Through point O, draw a line EOF. Which parallel to line AB.



Now $\because$ $AB \parallel EF$

$\angle OAP + \angle OAB = 180°$ (Linear pair)

$\angle OAP + 108° = 108°$

$\angle OAP = 180 - 108 = 72°$

$\therefore$ $\angle FOA = 72°$

Now $EF \parallel CD$ ($\because AB \parallel CD$)

$\because$ $\angle OCQ + \angle OCD = 180$ (Linear pair)

$\therefore$ $\angle OCQ + 120° = 180$

$\angle OCQ = 180 - 120 = 60°$

$\because$ $LOQ = \angle COF$ ($\because$ Alternate angle)

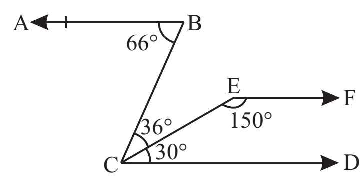
$\therefore$ $\angle COF = 60°$

then $\angle AOC = \angle AOF + \angle COF$

$\therefore$ $\angle AOC = 72° + 60°$

$\angle AOC = 132°$.

11. $\angle ABC = 66°$ (Given)



Now $\angle BCD = \angle BCE + \angle ECD$

$\angle BCD = 36 + 30 = 66°$

$\therefore$ $\angle BCD = 66°$

$\because$ $\angle ABC = \angle BCD = 66°$

So, $\angle ABC$ and $\angle BCD$ are alternate angle.

Since, $\angle ABC = \angle BCD$, i.e. alternate angles are equal.

So, $AB \parallel CD$ and BC is transversal line.

Now, increase the line EF to P and CD to M

then $\angle PEC + \angle CEF = 180°$ (Linear pair)

$\angle PEC + 150 = 180°$ ($\because \angle CEF = 150°$ Given)

$\angle PEC = 30°$

$\angle PEC = 30 = \angle ECD$

So, $\angle PEC$ and $\angle ECD$ are alternate angles.

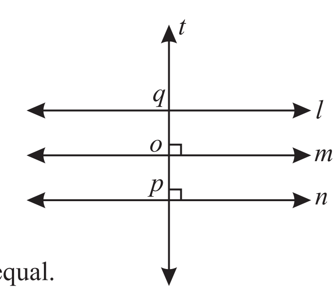
Since $\angle PEC = \angle ECD$ i.e. alternate angles are equal

So, $EF \parallel CD$

$\therefore$ $AB \parallel CD$ and $EF \parallel CD$

Hence, $AB \parallel EF$ **Proved.**

12. (a) $\because$ $t \perp m$



$\therefore$ $\angle tom = 90°$ in the same way,

$\because$ $t \perp n$

$\therefore$ $\angle tpn = 90°$

$\therefore$ $\angle tom = \angle tpn = 90$

So, $\angle tom$ and $\angle tpn$ are corresponding angles.

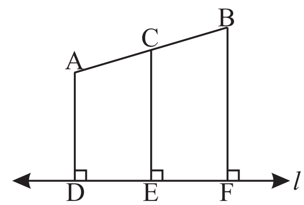
Since, $\angle tom = \angle tpn$, i.e. corresponding angles are equal.

So, $m || n$.
Now $\because$ $m || n$ and $l || m$
Hence, $l || n$. **Proved.**

(b) $\because$ $l || m$
$\therefore$ $\angle mot = \angle lqt$ (Corresponding angle)
$\therefore$ $\angle lot = 90°$ ($\because \angle mot = 90°$)
Hence $t \perp l$ **Proved.**

13. (a) True (b) False (c) False (d) False (e) False (f) True.

Exercise 8.2

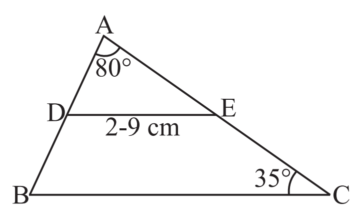


1. (a) $\because$ $AD \perp l$
$\therefore$ $\angle ADE = 90°$
then $\because$ $CE \perp l$
$\therefore$ $\angle CEF = 90°$
$\because$ $\angle ADE = \angle CEF = 90°$
Here, $\angle ADE$ and $\angle CEF$ are corresponding angles.
Since $AD || CE$ **Proved.**

(b) $\because$ $AD \perp l$ and $BF \perp l$
Hence $\angle ADl = \angle BFl = 90°$
So, $\angle ADl$ and $\angle BFl$ are corresponding angle
Since, $\angle ADl = \angle CEf$, i.e. corresponding angles are equal.
So, $AD || BF$ **Proved.**

(c) $\because$ $AD || CE$ and $A || BF$
Hence $CE || BF$
So, we can say $AD || CE || BF$ **Proved.**

(d) $AD || CD || FB$
$\therefore$ $\frac{AC}{CB} = \frac{DE}{EDF}$ (From equal intercept property) …(i)
$\because$ C is the mid point of AB
$\therefore$ $AC = CB$
or $\frac{AC}{CB} = 1$ …(ii)
From equations, (i) and (ii)
$\frac{DE}{EF} = 1$ or $DE = EF$
Thus, E is the midpoint of DF.



2. Given : D is M.P. of AB
$\Rightarrow$ $AD = DB = \frac{AB}{2}$
E is M.P. of AC
$\Rightarrow$ $AE = EC = \frac{AC}{2}$
$\angle A = 80, \angle C = 35°, DE = 2.9\,\text{cm}$
$\therefore$ In ΔABC, a line $DE || BC$, intersects AB in D and AC in E, then
$DE = \frac{BC}{2}$ (by proportional intercept theorem)

$$2.9 = \frac{BC}{2} \quad \Rightarrow \quad BC = 2 \times 2.9 = 5.8\,\text{cm}.$$

Now, $DE \parallel BC$ and AC is a transversal

$\therefore \quad \angle AED = \angle ACB = 35°$ (corresponding angles)

In $\angle ADE$, we know that $\angle A = 80, \angle E = 35°$

$\therefore \quad \angle A + \angle D + \angle E = 180$ (sum of all the angles of a triangle is 180°)

$80° + \angle D + 35° = 180°$

$\angle D = 180 - 115 = 65°$

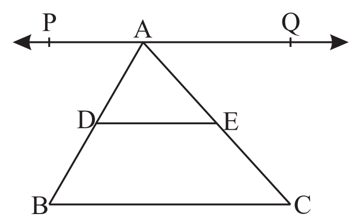
Now, $\angle ADE + \angle EDB = 180°$ (by linear pair)

$\Rightarrow \quad 65° + \angle EDB = 180°$

$\Rightarrow \quad EDB = 180° - 65° = 115°.$

3. Given : $PAQ \parallel DE \parallel BC$

$AD = 4$ cm, $DB = 8$ cm $EC = 10$ cm, $AE = ?$



$\therefore$ From equal intercept property

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{4}{8} = \frac{AE}{10}$$

$$\Rightarrow \quad AE = \frac{4 \times 10}{8} = 5\,\text{cm}.$$

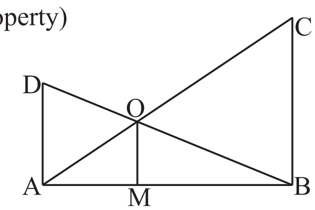
4. (a) Given : $AD \perp AB, OM \perp AB, CB \perp AB$ $OA = 2.4$ cm, $OC = 3.6$ cm.

In ΔABC, $OM \parallel BC$, intersects AB at M and AC at O.

$$\therefore \quad \frac{AM}{BM} = \frac{AO}{OC} \quad \text{(from equal intercept property)}$$

$$= \frac{2.4}{3.6}$$

$$= \frac{24}{36} = \frac{2}{3}.$$



(b) If $BO = 3$ cm Then In $\Delta ABD, OM \parallel DA$

$$\therefore \quad \frac{BO}{DO} = \frac{BM}{AM} \quad \text{(Form equal intercept property)}$$

$$\frac{3}{DO} = \frac{3}{2} \quad \text{[from part (a) above]}$$

$$DO = \frac{2 \times 3}{2} = 2\,\text{cm}.$$

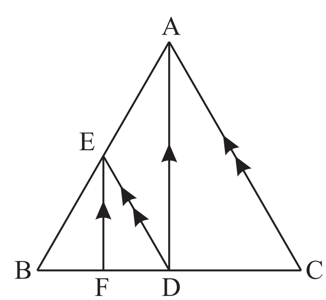
5. Given : $EF \parallel AD$ and $ED \parallel AC$ and $BF = 2$ cm, $FD = 3$ cm, $BE = 4$ cm,

$BD = BF + FD = 2 + 3 = 5$ cm.

In ΔABC, $ED \parallel AC$

$$\therefore \quad \frac{BE}{EA} = \frac{BD}{DC} \quad \Rightarrow \quad \frac{4}{EA} = \frac{5}{DC}$$

$$\Rightarrow \quad EA = \frac{4DC}{5} \quad \ldots(1)$$



In ABD, $EF \parallel AD$

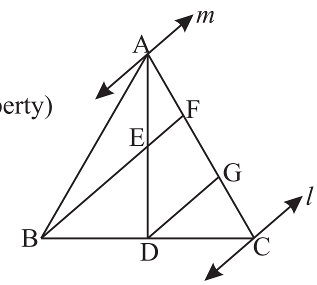
$$\therefore \quad \frac{BE}{EA} = \frac{BF}{FD} \quad \Rightarrow \quad \frac{4}{EA} = \frac{2}{3}$$

$\Rightarrow \quad EA = \dfrac{4 \times 3}{2} = 6\,\text{cm.}$ …(2)

From (1) & (2), we get

$6 = \dfrac{4}{5}DC \quad \Rightarrow \quad DC = \dfrac{6 \times 5}{4} = 7.5\ \text{cm.}$

$\therefore \quad BC = BD + DC = 5\,\text{cm} + 7.5\ \text{cm} = 12.5\ \text{cm.}$



6. (a) $\because \quad BF \parallel l$ Given

BC and FC are transversal lines

$\therefore \quad \dfrac{BD}{DC} = \dfrac{FG}{GC}$ (From equal intercept property)

$\because \quad BD = DC$ (Given)

$\therefore \quad \dfrac{FG}{GC} = 1$

or $FG = GC$ **Proved.** …(i)

(b) $\because M \parallel DG$ and AG and AD are transversal lines.

$\therefore \quad \dfrac{AF}{ED} = \dfrac{AF}{FG}$ (From equal intercept property)

$\because \quad AE = ED$ (Given)

$1 = \dfrac{AF}{FG}$ …(ii)

$\therefore \quad FG = AF$ **Proved.**

(c) $AC = 4.5$ cm (Given)

$AC = AF + FG + GC$

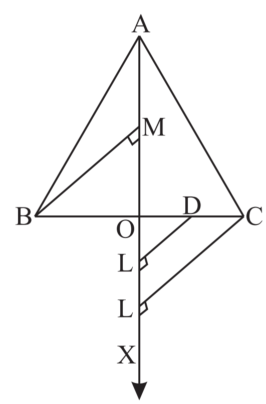
$\because \quad FF = FG = GC$ (from equation (i) and (ii))

$AC = AF + AF + AF$

$3AF = AC\ 3AF = 4.5\,\text{cm}$

$AF = \dfrac{4.5}{3} = 1.5$ cm

Hence, $AF = 1.5\,\text{cm}$.



7. (a) $\because \quad BM \perp AX$ (Given)

$LD \perp AX$ (Given)

$NC \perp AX$ (Given)

$\therefore \quad BM \parallel LD \parallel NC$ …(i)

Hence AX is a transversal for BM, LD and NC.

(b) $\because \quad BM \parallel NC \parallel LD$ (from equation (i))

$\therefore$ BC is transversal for BM, NC and LD.

(c) $\because$ BM, LD and NC are perpendicular to AX

So, these BM, LD and NC are parallel to each other.

(d) $ML = LN$

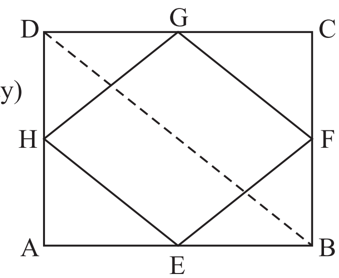
Yes.

8. Join DB.

Now, $AE = EB$ (Given)

and $DH = HA$ (Given)

Hence $\dfrac{AE}{EB} = 1$ and $\dfrac{DH}{HA} = 1$



So, $\frac{DH}{HA} = \frac{AE}{ED}$

$\because$ $\frac{DH}{HA} = \frac{AE}{EB}$ (From equal intercept property)

$\therefore$ $DB \parallel HE$.

in the same way

$DG = GC$ (Given)

$\therefore$ $\frac{DG}{GC} = 1$

and $FC = FB$ (Given)

$\therefore$ $\frac{FC}{FB} = 1$

Hence $\frac{DG}{GC} = \frac{FC}{FB}$

It shows the equal intercept property.

Hence $GF \parallel DB$

if $HE \parallel DB$ and $DB \parallel GF$

So, $GF \parallel HF$.

in the same way join A and C

Now, $\because$ $AE = FB$ (Given)

So, $\frac{AE}{EB} = 1$ and $FB = FC$

$\frac{FB}{FC} = 1$

So, $\frac{AE}{EB} = \frac{FB}{FC}$

It shows the equal intercept property of parallel lines

Hence $AC \parallel EF$

No $\because$ $DG = GC$ (Given)

So, $\frac{DG}{GC} = 1$ and $\frac{DH}{HA} = 1$

Now, $\frac{DG}{GC} = \frac{DH}{HA}$

It is also represent the equal incept propertyof parallel lines.

So, $AC \parallel HG$

$\because$ $AC \parallel EF$

and $AC \parallel HG$

then $EF \parallel HG$

We know that in parallelogram opposite sides are parallel.

In $EFGH$, $HE \parallel GF$ and $EF \parallel GH$

Hence, $EFGH$ is a parallelogram. **Proved.**

MCQ's

1.(c) **2.** (b) **3.** (b) **4.** (c) **5.** (c).