Exercise = 5.3

1. (i) 92, (iii) 720, (v) 2398, (vi) 179832,
(vii) 468230, (ix) 379514

2. (ii) 78, (iv) 1693, (v) 267144, (vii) 1248964

3. (i) 30 (iii) 75 (iv) 210 (v) 305 (vi) 640
(viii) 985 (ix) 10775 (x) 24375

4. (ii) 180 (v) 855 (vii) 990 (ix) 801801 (x) 225225

5. (i) 90 (ii) 150 (iii) 500 (iv) 880 (vi) 18930
(viii) 711980 (ix) 46950

6. (ii) 501 (iii) 213 (iv) 102 (vi) 426

7. (i) 3 (ii) 9

(i) 735	(i) 765
(ii) 312	(ii) 342
(iii) 171	(iii) 171
(iv) 141	(iv) 144
(v) 222	(v) 252

8. 273 x 6
$2 + 7 + 3 + x + 6 = 18 + x$ $x = 0$

9. 31 x 26
$3 + 1 + x + 2 + 6 = 12 + x$ $x = 0$

10. 712 x 6
$7 + 1 + 2 + x + 6 = 16 + x$ x = 2, 11, ___

11. $(1000a + 100b + 10c + d) - (a + b + c + d)$
$= 999a + 99b + 9c = 9 (111a + 11b + c)$ proved

12. 4x y 4 x
$4 + x + y + 4 + x = 8 + 2x + y$ x = 5 (divisible by 5)
$= 8 + 2 \times 5 + y = 8 + 10 + y$ y = 3

Chapter-6 Algebraic Expressions and Identities

Exercise = 6.1

1. (i) 4xyz, – 3xy (ii) y, y^3
(iii) $3x^2y^2, -7x^2y^2z^2, y^2$ (iv) – ab, bc, – ca
(v) x (vi) 0.7p, 0.8pq, 0.8r

2. (i) 11xy, – 2yx Yes (ii) $9x^2y, -17xy^2$ No
(iii) $10a^2b^2c, -17cb^2a^2$ Yes

3. (i) 9x + 11 Yes (ii) $\frac{4y^2+3y+7}{y}$ No
(iii) $\frac{1}{5} \neq 12x^2 + \frac{7}{2}x^4 \frac{-1}{9}x^3$ Yes

4. (i) degree = 3 (ii) degree = 4 (iii) degree = 5 (iv) degree = 6

5. (i) $9ab - 6ab + 4ab - 2ab = 13ab - 8ab = 5ab$

(ii) $8x - 4x + 6x - 2x - 3x = 14x - 9x = 5x$

(iii) $3a - 4b + 4c + 2a + 3b - 8c + a - 6b + c = 6a - 7b - 3c$

(iv) $5x + 8y + 2z + 3z - 4y - 2x + 6y - z - x + 3x - 2z - 3y$

$= (5x - 2x - x + 3x) + (8y - 4y + 6y - 3y) + 2z + 3z - z - 2z$

$= 5x + 7y + 2z$

(v) $6ax - 2by + 3cz + 6by - 11ax - cz + 10cz - 2ax - 3by$

$= -7ax + by + 12cz$

(vi) $2x^3 - 9x^2 + 8 + 3x^2 - 6x - 5 + 7x^3 - 10x + 1 + 3 + 2x - 5x^2 - 4x^3$

$= 5x^3 - 11x^2 - 14x + 7$

(vii) $6p + 4q - r + 3 + 2r - 5p - 6 + 11q - 7p + 2r - 1 + 2q - 3r + y$

$= -6p + 17q$

(viii) $4x^2 - 7xy + 4y^2 - 3 + 5 + 6y^2 - 8xy + x^2 + 6 - 2xy + 2x^2 - 5y$

$= 7x^2 - 17xy + 5y^2 + 8$

6. (i) $8x - 17 - (3x - 5)$

$= 8x - 17 - 3x + 5 = 5x - 12$

(ii) $5x^2 11x + 19 - (2x^2 - 5x + 10)$

$= 5x^2 - 11x + 19 - (2x^2 - 5x + 10) = 3x^2 - 6x + 9$

(iii) $2p - 3q - 7r - 11 - (-7p + 2q - 3r + 5)$

$= 2p - 3q - 7r - 11 + 7p - 2q + 3r - 5 = 9p - 5q - 4r - 16$

(iv) $25x^2 + 16xy - 3b^2 - 2 - (13xy - 6x^2 + 4a^2 - 1)$

$= 25x^2 + 16xy - 3b^2 - 2 - 13xy + 6x^2 - 4a^2 + 1 = 31x^2 + 3xy - 3b^2 - 4a^2 - 1$

7. $2x^2 - 9x + 1 - (-4x^2 + 3x + 1)$

$= 2x^2 - 9x + 1 + 4x^2 - 3x - 1 = 6x^2 - 12x$

8. $3r^2 p - 2pq^2 + qr^2 - (2r^2 p + 4pq^2 - 3qr^2)$

$= 3r^2 p - 2pq^2 + qr^2 - 2rp - 4pq^2 + 3qr^2 = r^2 p - 6pq^2 + 4qr^2$

9. Perimeter $= 2(3x^2 - 5y^2 + 7x^2 - xy)$

$= 2(10x^2 - 5y^2 - xy) = 20x^2 - 10y^2 - 2xy$

10. Third side $= (7p^2 - 8p + 9) - (2p^2 - p + 1 + 11p^2 - 3p + 5)$

$= 7p^2 - 8p + 9 + (13p^2 - 4p + 6) = 7p^2 - 8p + 9 - 13p^2 + 4p - 6 = -6p^2 - 4p + 3$

Exercise = 6.2

1. (i) $a^2b^2 \times -3 = -3a^2b^2$

(ii) $4mn \times (-1)^7 p^2 q = -4mnp^2 q$

(iii) $\frac{-2}{7} a^2 c \times \frac{16}{21} bd^2 = -\frac{32}{147} a^2 bcd^2$

(iv) $\left(\frac{-17}{8} x^3 y^2\right) \times \left(\frac{-16^2}{51_3} x^2 y\right) = \frac{2}{3} x^5 y^3$

(v) $\left(\frac{-4}{13} x\right)\left(\frac{65^5}{16_4} x^2 y\right)(-4) = 5x^3 y$

(vi) $(-2xy)(-3x^3 y^2)\left(\frac{-1}{6} x^2 y^7\right) = x^6 y^{10}$

2. (i) $2a(3a^2 + 5a - 1) = 6a^2 + 10a^2 - 2a$

(ii) $-3a^2 b(2a^2 - 3b^2 + ab) = -6a^2 b + 9a^2 b^3 - 3a^3 b^2$

(iii) $4xy(9x^2 y - 2xy^3 + xy) = 36x^3 y^2 - 8x^2 y^4 + 4x^2 y^2$

(iv) $2xyz(x^2y^2 + y^2z^2 - 2yz) = 2x^2y^3z + 2xy^3z^3 - 4xyz^2$

3. (i) $(a^2 - b^2)(a^2 + b^2)$

$= a^2(a^2 + b^2) - b^2(a^2 + b^2) = a^4 + a^2b^2 - a^2b^2 - b^4 = a^4 - b^4$

(ii) $(5x^2 + 2y^2)\ (3x^2 - 7y^2)$

$= 5x^2(3x^2 - 7y^2) + 2y^2(3x^2 - 7y^2)$

$= 15x^4 - 35x^2y^2 + 6x^2y^2 - 14y^4 = 15x^2 - 29x^2y^2 - 14y^4$

(iii) $\left(3x + \frac{5y}{7}\right)\left(2x - \frac{3y}{7}\right)$

$= 3x\left(2x - \frac{3y}{7}\right) + \frac{5y}{7}\left(2x - \frac{3y}{7}\right)$

$= 6x^2 - \frac{9xy}{7} + \frac{10}{7}xy - \frac{15}{49}y^2 = 6x^2 - \frac{1}{7}xy - \frac{15}{49}y^2$

(iv) $\left(\frac{1}{7}x - \frac{1}{8}y\right)\left(7x^2 - 8y^2\right)$

$= \frac{1}{7}x\left(7x^2 - 8y^2\right) - \frac{1}{8}y\left(7x^2 - 8y^2\right) = x^3 - \frac{8}{7}xy^2 - \frac{7}{8}x^2y + y^3$

(v) $(m^3 - n^3)(m^2 + n^2)$

$= m^3(m^2 + n^2) - n^3(m^2 + n^2) = m^5 + m^3n^2 - n^3m^2 - n^5$

(vi) $(a^2 + b^2)\ (a^2 - b^2)$

$= a^2(a^2 - b^2) + b^4(a^2 - b^2)$

$= a^4 - a^2b^2 + a^2b^4 - b^4$

4. (i) $(x^2 - 5x + 8) \times (x^2 + 2)$

$= x^2(x^2 - 5x + 8) + 2(x^2 - 5x + 8)$

$= x^4 - 5x^3 + 8x^2 + 2x^2 - 10x + 16$

$= x^4 - 5x^3 + 10x^2 - 10x + 10$

(ii) $(x^3 - 5x^2 + 3x + 1) \times (x^2 - 3)$

$= x(x^3 - 5x^2 + 3x + 1) - 3(x^3 - 5x^2 + 3x + 1)$

$= x^4 - 5x^3 + 3x^2 + x - 3x^3 + 15x^2 - 9x - 3$

$= x^4 - 8x^3 + 18x^2 - 8x - 3$

(iii) $(3x + 2y - 4) \times (x - y + 2)$

$= 3x(x - y + 2) + 2y(x - y + 2) - 4(x - y + 2)$

$= 3x^2 - 3xy + 6x + 2xy - 2y^2 + 4y - 4x + 4y - 8$

$= 3x^2 - 2y^2 - xy + 2x + 8y - 8$

(iv) $(x^2 - 5x + 8)(x^2 + 2x - 3)$

$= x^2(x^2 + 2x - 3) - 5x(x^2 + 2x - 3) + 8(x^2 + 2x - 3)$

$= x^4 + 2x^3 - 3x^2 - 5x^3 - 10x^2 + 15x + 8x^2 + 16x - 24$

$= x^4 - 3x^3 - 2x^2 + 31x - 24$

(v) $(2x^2 + 3x - 7) \times (3x^2 - 5x + 4)$

$= 2x^2(3x^2 - 5x + 4) + 3x(3x^2 - 5x + 4) - (3x^2 - 5x + 4)$

$= 6x^4 - 10x^3 + 8x^2 + 9x^3 - 15x^2 + 12x - 21x^2 + 35x - 28$

$= 6x^4 - x^3 - 28x^2 + 47x - 28$

(vi) $(9x^2 - x + 15) \times (x^2 - x - 1)$

$= 9x^2(x^2 - x - 1) - x(x^2 - x - 1) + 15(x^2 - x - 1)$

$= 9x^4 - 9x^3 - 9x^2 - x^3 + x^2 + x + 15x^2 - 15x - 15$

$= 9x^2 - 10x^3 + 7x^2 - 14x - 15$

5. (i) $3x\,(4x - 5) + 3$

$= 12x^2 - 15x + 3$

x = 1	$x = \frac{1}{2}$
$= 12 \times 1^2 - 15 \times 1 + 3$	$= 12 \times \frac{1}{4} - 15 \times \frac{1}{2} + 3$
$= 12 - 15 + 3$	$= 3 - \frac{15}{2} + 3$
$= 15 - 15$	$= \frac{12 - 15}{2}$
$= 0$	$= \frac{-3}{2}$

(ii) $9(a^2 + a + 1) + 5$

a = –1

$= 9a^2 + 9a^2 + 9 + 5 \quad = 9a^2 + 9a + 14$

a = – 1

$= 9(-1)^2 + 9(-1) + 14 \quad = 9 - 9 + 14 = 14$

(iii) $7x(x^2 - x + 1)$

$= 7x^3 - 7x^2 + 7x$	x = 2
x = – 1	$7(2)^3 - 7(2)^2 + 7x$
$= 7(-1)^3 - 7(-1)^2 + 7(-1)$	= 56 – 28 + 14
= – 7 – 7 – 7	= 56 – 14
= – 21	= 42

(iv) $3y^2(3y + 3) + 53$ y = – 1

$= 9y^3 + 9y^2 + 53$

y = – 1

$= 9(-1)^3 + 9(-1)^2 + 53$

$= -9 + 9 + 53 = 53$

(v) $x^3 y^3 \left[\frac{-3}{2}(x - y)\right]$ x = 0, y = 1

$= \frac{-3}{2} x^4 y^3 + \frac{3}{2} x^3 y^4$

x = 0, y = 1

= 0

6. (i) $x^3 y - 2x^2 y(3x + 2y) + 5x^2 y^2$

$= x^3 y - 6x^3 y - 4x^2 y^2 + 5x^2 y^2 = -5x^3 y + x^2 y^2$

(ii) $3x(2x^2 - x + 1) - 2x^2(5x + 3)$

$= 6x^3 - 3x^2 + 3x - 10x^2 - 6x^2 = 14x^3 - 3x^2 + 3x$

(iii) $2a^2(a^3-a)-3a(a^4+2a)-2(a^4-3a^{2)}$

$= 2a^5-2a^3-3a^5-6a^2-2a^4+6a^2$

$= -a^5-2a^4-2a^3$

(iv) $x^2y(x^3-x+1)-xy(x^4-2x^2+2x)-y(x^3-x-1)$

$= \cancel{x^5y}-\cancel{x^3y}+n^2y-\cancel{n^5y}+\cancel{2x^3y}-2x^2y-\cancel{x^3y}+xy+y$

$= 3x^2y+xy+y$

(v) $(x+2y)(1+3x+4y)-6y(x+y)$

$= x+3x^2+4xy+2y+\cancel{6xy}+8y^2-\cancel{6xy}-6y^2$

$= 3x^2+2y^2+4xy+x+2y$

(vi) $x^2+(3x-y)\ (3x+y+y^2)$

$= x^2+9x^2+3y+3xy^2-3y-y^2-y^3$

$= 10x^2+3xy^2-y^2-y^3$

7. (i) $(a+b)(a^2-ab+b^2)$

$= a^3-a^2b+\cancel{ab^2}+\cancel{a^2b}+\cancel{a^2b}-\cancel{ab^2}+b^2$

$= a^3+b^3$

RHS a = 1, b = 1

$1^3+1^3 = 1+1 = 2$

LHS $(1+1)(1^2-1\cdot1+1^2) = 1\times1 = 2$

LHS = RHS

(ii) $a^2+(3a-b)(3a+b+c^2)$

$= a^2+9a^2+3b+3ac^2-3b-b^2-bc^2$

$= 10a^2+3a^2-bc^2-b^2$

RHS a = 1, b = 1, c = 2

$= 10\times1+3\times1\times4-1\times4-1$

$= 10+12-4-1 = 22-5 = 17$

LHS $1^2+(3\times1-1)\ (3\times1+1+4)$

$= 1+2\times8 = 1+16 = 17$

LHS = RHS

(iii) $a(a+b^2+c)+b^2(a+b+c)-c(a+b^2)$

$= a^2+ab^2+ab^2+b^3+c$

$= a^2+2ab^2+b^3$

LHS 1 (1 + 1 + 2) + 1 (1 + 1 + 2) – 2 (1 + 1)

$= 4+4-2\times2 = 4$

RHS $1+2\times1\times1+1$

$= 1+2+1 = 4$

LHS = RHS

(iv) $(a^2+b^2+c^2)a^3+(a^3-b^3-c^3)b^3$

$= a^5+a^3b^2+a^3c^2+a^3b^3-b^6-b^3c^3$

LHS (1 + 1 + 4) 1 + (1 – 1 – 8) 1

$= 6+(-8) = -2$

LHS = RHS

Exercise = 6.3

1. (i) $\frac{{}^{3}9x^3y^2z^2}{3x^2yz} = 3xyz$ (ii) $\frac{{}^{5}20m^2n^2}{4m^2n^2} = 5n$

(iii) $\frac{{}^{4}32a^3b^3}{8ab} = -4a^2b^2$ (iv) $\frac{{}^{3}-21abc^2}{7abc} = -3c$

(v) $\frac{{}^{8}72xyz^2}{-9xz} = -8yz$ (vi) $\frac{{}^{9}-81a^4b^5c^8}{-9a^2b^2c^3} = 9a^2b^3c^5$

(vii) $\frac{{}^{4}16m^3y^2}{4m^3y} = 4y$ (viii) $\frac{{}^{8}32m^2n^3p^2}{4m^2n^2p^2} = 8n$

2. (i) $\frac{12m^5+15m^4-9m^2}{3m^2} = \frac{3m^2(4m^3+5m^2-3)}{3m^2} = 4m^3+5m^2-3$

(ii) $\frac{4x^3y+2x^2y^2-4xy}{2xy} = \frac{2xy(2x^2+xy-2)}{2xy} = 2x^2+xy-2$]

(iii) $\frac{8x^4yz-4xy^3z+8x^2yz^4}{2xyz} = \frac{2xyz(4x^3-2y^2+4xz^3)}{xyz} = 4x^3-2y^2+4xz^3$

(iv) $\frac{\frac{4}{5}a^2b^2c^2+\frac{3}{4}ab^2c^3-\frac{1}{6}ab^3c^3}{\frac{1}{2}abc} = \frac{\frac{1}{2}abc\left(\frac{8}{5}abc+\frac{3}{2}bc^2-\frac{1}{3}b^2c^2\right)}{\frac{1}{2}abc}$

$= \frac{8}{5}abc+\frac{3}{2}bc^2-\frac{1}{3}b^2c^2$

(v) $\frac{9x^2y-6xy+12xy^2}{\frac{-3}{2}xy} = \frac{-3xy\times 2(3x-2+4y)}{3xy} = -6x+4-8y$

(vi) $\frac{4z^3+6z^2-z}{\frac{-1}{2}z} = \frac{z(4z^2+6z-1)}{\frac{-1}{2}z} = 2(4z^2+6z-1) = -8z^2-12z+2$

3. (i)

$$\begin{array}{r|l} & 2x-5 \\ 2x+2 & \overline{)\,4x^2-6x-10\,(} \\ & \underline{-4x^2+4x} \\ & -10x-10 \\ & \underline{-10x-10} \\ & \times \end{array}$$

(ii)

$$\begin{array}{r|l} & x-23/5 \\ 5x+11 & \overline{)\,5x^2-34x-99\,(} \\ & \underline{-5x^2-11x} \\ & -23x-99 \\ & \underline{-23x-\frac{235}{5}} \\ & \frac{253}{5}-99 \\ & \underline{\frac{253-495}{5}} \\ & \frac{242}{5} \end{array}$$

(iii)
$$\begin{array}{r|l} & 2x-7 \\ \hline 2x-13 & 4x^2-40x+91 \\ & 4x^2-26x \\ & \hline & -14x+91 \\ & -14x+91 \\ & \hline & \times \end{array}$$

(iv)
$$\begin{array}{r|l} & 9x+1 \\ \hline x+1 & 9x^2+10x+1 \\ & 9x^2+9x \\ & \hline & x+1 \\ & x+1 \\ & \hline & \times \end{array}$$

(v)
$$\begin{array}{r|l} & x^3+x^2-9x+18 \\ \hline x+1 & x^4+2x^3-8x^2+9x+18 \\ & x^4+x^3 \\ & \hline & x^3-8x^2 \\ & x^3+x^2 \\ & \hline & -9x^2+9x \\ & -9x^2-9x \\ & \hline & 18x+18 \\ & 18x+18 \\ & \hline & \times \end{array}$$

(vi)
$$\begin{array}{r|l} & -3m+3 \\ \hline 4m^3-2m+1 & -12m^4+12m^3+16m^2-11m+4 \\ & -12m^4+6m^2-3m \\ & \hline & 12m^3+10m^2-8m+4 \\ & 12m^3-6m+3 \\ & \hline & 10m^2-2m+1 \end{array}$$

Reminder

4. (i) Divident = Division × quotient + Remainder

$6p^3+5p^2+4=(2p+1)(3p^2+p)-p+4$

$=6p^3+2p^2+3p^2+p-p+4$

$=6p^3+5p^2+4$

LHS = RHS

$$\begin{array}{r|l} & 3p^2+p \\ \hline 2p+1 & 6p^3+5p^2+4 \\ & 6p^3+3p^2 \\ & \hline & 2p^2+4 \\ & 2p^2+p \\ & \hline & -p+4 \end{array}$$

(ii)
$$\begin{array}{r|l} & x^2-5x+29 \\ \hline x+5 & x^3+4x-3 \\ & x^3+5x^2 \\ & \hline & -5x^2+4x \\ & -5x^2-25x \\ & \hline & 29x-3 \\ & 29x+145 \\ & \hline & -148 \end{array}$$

$x^3+4x-3=(x+5)(x^2-5x+29)-148$

$=x^3-5x^2+29x+5x^2-25x+148$

$=x^3+4x-3$

$$\begin{array}{r|l} & 3x^2+2x-1 \\ \hline 3x-1 & 9x^3+3x^2-5x+7 \\ & 9x^3-3x^2 \\ & \hline & 6x^2-5x \\ & 6x^2-2x \\ & \hline & -3x+7 \\ & -3x+1 \\ & \hline & 6 \end{array}$$

(iii) $ax^3+3x^2-5x+7=(3x-1)(3x-1)(3x^2+2x-1)+6$

$=9x^3+6x^2-3x-3x^2-2x+1+6$

$=9x^3+3x^2-5x+7$

(iv) $3x^5-2x^4+x^2-2=(x^2+x+1)$

$(3x^2+5x^2-8x)+4x^2+8x-1$

$3x^5+5x^4-8x^3+3x^4+5x^3-8x^2+3x^3+$

$5x^2-8x^2+4x^2+5x^2-8x+4x^2+8x-2$

$=3x^5-2x^4+x^2-2$

$$\begin{array}{r|l} & 3x^3+5x^2-8x \\ x^2+x+1 & \overline{3x^5-2x^4+x^2-2} \\ & 3x^5+3x^4+3x^3 \\ & \hline & 5x^4-3x^3+x^2-2 \\ & 5x^2+5x^3+5x^2 \\ & \hline & -8x^3-4x^2-2 \\ & -8x^3-8x^2-8x \\ & \hline & 4x^2+8x-2 \end{array}$$

5.

$$\begin{array}{r|l} & x^2-4x+7 \\ x^2+3x+2 & \overline{x^4-x^3-3x^2+x+2} \\ & x^4+3x^3+2x^2 \\ & \hline & -4x^3-5x^2+x+2 \\ & -4x^3-12x^2-84 \\ & \hline & 7x^2+9x+2 \\ & 7x^2+21x+14 \\ & \hline & -12x-12 \end{array}$$

6.

$$\begin{array}{r|l} & z^2+3 \\ 4z^2-5 & \overline{4z^4+17z^2+15} \\ & 4z^4-5z^2 \\ & \hline & 12z^2+15 \\ & 12z^2-15 \\ & \hline & \times \end{array}$$

7.

$$\begin{array}{r|l} & x^2+3x+2 \\ x^2+3x+2 & \overline{x^4+6x^3+13x^2+13x+8} \\ & x^4+3x^3+2x^2 \\ & \hline & 3x^3+11x^2+13x+8 \\ & 3x^3+9x^2+6x \\ & \hline & 2x^2+7x+8 \\ & 2x^2+6x+4 \\ & \hline & x+4 \end{array}$$

8.

$$\begin{array}{r|l} & 3x^3+2x^2-9x-1/2 \\ 2x^2-3 & \overline{6x^5+4x^4-27x^3-7x^2-27x-6} \\ & 6x^5-9x^3 \\ & \hline & 4x^4-18x^3-7x^2-27x-6 \\ & 4x^4-6x^2 \\ & \hline & -18x^3-x^2-27x-6 \\ & -18x^3-27x \\ & \hline & -x^2-54x-6 \\ & -x^2+\frac{3}{2} \\ & \hline & 54x-6\frac{-3}{2} \\ & 54x\frac{-15}{2} \end{array}$$

9. a – 1 = 0

a = 1

b = 7

$$\begin{array}{r|l} & x^2+x+7 \\ x^2+1 & \overline{x^4+x^3+8x^2+9x+b} \\ & x^4+x^2 \\ & \hline & x^3+7x^2 \\ & x^3+x \\ & \hline & 7x^2+(a-1)x \\ & 7x^2+7 \\ & \hline & (a-1)x+b-7 \end{array}$$

10. $t^3-2t^2+3t-18$ is not divisible by -3

11.

$$
\begin{array}{r|l}
x^2-3x+4 & 5x+2 \\
\hline
 & 5x^3-13x^2+15x+7 \\
 & 5x^3-15x^2+20x \\
 & \quad -\quad +\quad - \\
\hline
 & \qquad 2x^2-5x+7 \\
 & \qquad 2x^2-6x+8 \\
 & \qquad -\quad +\quad - \\
\hline
 & \qquad\qquad x-1
\end{array}
$$

Exercise = 6.4

1. (i) $(x+2)(x+2)$

$=(x+2)^2=x^2+2^2+2\times x\times 2=x^2+4+4x$

(ii) $(3x+5)(3x+5)$

$=(3x+5)^2=(3x)^2+5^2+2\times 3x\times 5=9x^2+25+30x$

(iii) $\left(\frac{3}{4}x^2+5\right)\left(\frac{3}{4}x^2+5\right)$

$=\left(\frac{3}{4}x^2+5\right)^2=\left(\frac{3}{4}x^2\right)^2+5^2+2\times\frac{3}{4^2}x^2\times 5=\frac{9}{16}x^4+25+\frac{15}{2}x^2$

(iv) $(y-9)(y-9)=(y-9)^2$

$=y^2+9^2-2\times y\times 9=y^2+81-18y$

(v) $\left(7t-\frac{1}{2}u\right)\left(7t-\frac{1}{2}u\right)$

$=(7t)^2+\left(\frac{1}{2}u\right)^2-2\times 7t\times\frac{1}{2}u=49t^2+\frac{1}{4}u^2-7tu$

(vi) $(5x^2-4y^2)(5x^2-4y^2)$

$=(5x^2)^2+(4y^2)^2-2\times 5x^2\times 4y^2$

$=25x^4+16y^4-40x^2y^2$

(vii) $\left(\frac{2}{3}x+\frac{4}{5}y\right)\left(\frac{2}{3}x+\frac{4}{5}y\right)$

$=\left(\frac{2}{3}x\right)^2+\left(\frac{4}{5}y\right)^2+2\times\frac{2}{3}x\times\frac{4}{5}y=\frac{4}{9}x^2+\frac{16}{5}y^2+\frac{16}{15}xy$

(viii) $\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$

$=\left(\frac{3}{4}x\right)^2+\left(\frac{5}{6}y\right)^2-2\times\frac{3}{4^2}x\times\frac{5}{6^2}y=\frac{9}{16}x^2+\frac{25}{36}y^2-\frac{5}{4}xy$

(ix) $\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$

$=\left(\frac{1}{2}y^2\right)^2+\left(\frac{1}{3}y\right)^2-2\times\frac{1}{2}y^2\times\frac{1}{3}y=\frac{1}{4}y^4+\frac{1}{9}y^2-\frac{1}{3}y^3$

2. (i) $(2x+3y)^2$

$= (2x)^2 + (3y)^2 + 2\times 2x + 3y = 4x^2 + 9y^2 + 12xy$

(ii) $(3xy+2z)^2$

$= (3xy)^2 + (2z)^2 + 2\times 3xy \times 2z = 9x^2y^2 + 4z^2 + 12xyz$

(iii) $\left(\frac{3}{4}m + \frac{2}{5}n\right)^2$

$= \left(\frac{3m}{4}\right)^2 + \left(\frac{2}{3}n\right)^2 + 2\times\frac{3}{4^2}m\times\frac{2}{5}n = \frac{9}{16}m^2 + \frac{4}{25}n^2 + \frac{3}{5}mn$

(iv) $(0\cdot 5x - 0\cdot 3y)^2$

$= (0\cdot 5x)^2 + (0\cdot 3y)^2 - 2\times 0\cdot 5x \times 0\cdot 3y = 0\cdot 25x^2 + 0\cdot 09y^2 - 3xy$

(v) $(7x^2 - 5y^2)^2$

$= (7x^2)^2 + (5y^2)^2 - 2\times 7x^2 \times 5y^2 = 49x^4 + 25y^4 - 70x^2y^2$

(vi) $(4p-3q)^2$

$= (4p)^2 + (3q)^2 - 2\times 4p\times 3q = 16p^2 + 9q^2 - 24pq$

(vii) $(4xy+5y)^2$

$= (4xy)^2 + (5y)^2 + 2\times 4xy\times 5y = (6x^2y^2 + 25y^2 + 40xy^2$

(viii) $(3xy-2z)^2$

$= (3xy)^2 + (2z)^2 - 2\times 2xy\times 2z = 9x^2y^2 + 4z^2 - 12xyz$

(ix) $(1\cdot 1p - 2\cdot 2a)^2$

$= (1\cdot 1p)^2 + (2\cdot 2q)^2 - 2\times 1\cdot 1p\times 2\cdot 2q = 1\cdot 21p + 4\cdot 41q^2 - 4\cdot 84pq$

3. (i) $(5x+3y)(5x-3y)$

$= (5x)^2 - (3y)^2 = 25x^2 - 9y^2$

(ii) $(y-1)(y+1)(y^2+1)(y^4+1)$

$= (y^2-1^2)(y^2+1)(y^4+1)$

$= (y^2-1)(y^2+1)(y^4+1)$

$= [(y^2)^2 - 1^2](y^4+1)$

$= (y^4-1)(y^4+1)$

$= y^8 - 1$

(iii) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left[(x^2)^2 - \left(\frac{1}{x^2}\right)^2\right]\left[x^4+\frac{1}{x^4}\right]$

$= \left(x^4-\frac{1}{x^4}\right)\left(x^4+\frac{1}{x^4}\right)$

$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2$

$= x - \frac{1}{x^8}$

(iv) $(3x+y)(3x-y)(ax^2+y^2)$

$=[(3x^2)-y^2](9x^2+y^2)=(9x^2-y^2)(9x^2+y^2)$

$=(9x^2)^2-(y^2)^2=81x^4-y^4$

4. (i) $(y+7)(y+5)$

$=y(y+5)+7(y+5)=y^2+5y+7y+35=y^2+12y+35$

(ii) $(p-3)(p-3)$

$=p^2+3^2-2\times p\times 3=p^2+9-6p$

(iii) $(2x+3)(2x-6)$

$=2x(2x-6)+3(2x-6)=4x^2-12x+6x-18=4x^2-6x+8$

(iv) $(2x-1)(2x-7)$

$=2x(2x-7)-1(2x-7)$

$=4x^2-14x-2x+7=4x^2-16x+7$

(v) $(7p^2\ 5)(7p^2+8)$

$=7p^2\ (7p^2+8)-5(7p^2+8)$

$=49p^2+56p^2-35p^2-40=49p^2+21p^2-40$

(vi) $(2x-3)(2x-5)$

$=2x(2x-5)-3(2x-5)$

$=4x^2-10x-64+15=4x^2-16x+15$

5. (i) 108×108

$=(100+8)(100+8)=100^2+8^2+2\times 100\times 8$

$=10000+64+1600=11664$

(ii) $(151)^2$

$=(150+1)^2=150^2+1^2+2\times 150\times 1$

$=22500+1+300=22801$

(iii) $(599)^2$

$=(600-1)^2=600^2+1^2-2\times 600\times 1$

$=360000+1-1200=361201$

(iv) $(0\cdot 97)^2$

$=(1-0\cdot 03)^2=1^2+0\cdot 03^2-2\times 1\times 0\cdot 03$

$=1+0\cdot 0009-0\cdot 06=0\cdot 9409$

(v) 992×1008

$=(1000-8)(1000+8)=(1000)^2-8^2$

$=1000000-64=999936$

(vi) 218×182

$=(200+18)(200-18)=(200)^2-18^2$

$=40000-324=39676$

(vii) $(199\cdot 6)^2$

$=(200-0\cdot 4)^2=200^2+0\cdot 4^2-2\times 200\times 0\cdot 4$

$=40000+0\cdot 16-160=39840\cdot 16$

(viii) $(395)^2$

$=(400-5)^2=400^2+5^2-2\times 400\times 5=160000+25-40000=156025$